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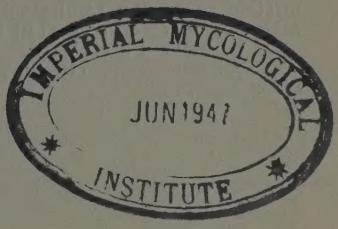

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MATHEMATICS

ON A GENERALIZED PROBLEM OF MOMENTS

By M. KREIN

(Communicated by S. N. Bernstein, Member of the Academy, 20.XI.1943)

The present paper contains the solution of the fundamental questions of the generalized problem of moments. This problem has arisen in connexion with the investigations of M. Livshitz(1), who obtained the first important result pertaining to this problem and disclosed its perspectives. A part of the results exposed follows immediately from our preceding papers(2).

1. In the sequel L denotes a linear set of analytic functions $f(z)$ engaging the following properties:

1°. Every function $f(z)\in L$ is holomorphic in a certain domain G_f containing the real axis.

2°. If the function $f(z)\in L$, the point $a\in L$ and $f(a)=0$, then the function $(z-a)^{-1}f(z)\in L$.

Let $B(g, f)$ $(g, f\in L)$ be a functional possessing the following properties:

I. $B(g, f)=\overline{B(f, g)}$ $(g, f\in L)$

II. $B(\lambda g+\mu h, f)=\lambda B(g, f)+\mu B(h, f)$ $(g, h, f\in L;$ λ, μ, complex numbers)

III. $B(g, g)\geqslant 0$ $(g\in L)$

IV. $B(zg, f)=B(g, zf)$ $(g, f\in D)$

where D denotes the set of all those $f\in L$, for which $zf(z)\in L$.

The properties I, II, III make it possible to conceive the functional B as a scalar product on L making L an incomplete Hilbert space L_B, if we identify any two elements $g, f\in L$, for which the distance

$$\|g-f\|=\sqrt{B(g-f, g-f)}=0$$

Definition 1. The functional $B(g, f)$ $(g, f\in L)$ is called *H-functional*, if the set D is dense in L_B.

Definition 2. A non-decreasing function $\sigma(t)=\sigma(t-0)$ $(-\infty<t<\infty$, $\sigma(0)=0)$ is called *distribution function* of the functional $B(g, f)$ $(g, f\in L)$ if

$$B(g, f)=\int_{-\infty}^{\infty} g(t)\,\overline{f(t)}\,d\sigma(t) \quad (g, f\in L) \tag{1}$$

Theorem 1. *Every H-functional $B(g, f)$ $(g, f\in L)$ possesses at least one distribution function.*

In the important case $1\in L$ Theorem 1 is a particular case of the theorem of M. Livshitz(1).

The arguments applied to prove the latter theorem enable us to establish Theorem 1 also in the case where $1\overline{\in}L$.

Theorem 2. *If the functional $B(g, f)$ $(g, f\in L)$ possesses more than one distribution function, then $B(g, g)>0$ for every function $g\in L$, not identically equal to zero.*

2. In the following two theorems we assume that $1\in L$. Note that if this condition is not satisfied, but L contains at least one function not vanishing anywhere on the real axis, then we achieve that this condition is satisfied simply by dividing all functions from L by this function.

Theorem 3. *If* $1 \in L$ *and the H-functional* $B(g, f)$ $(g, f \in L)$ *possesses more than one distribution function, then*

1°. *Every function* $f \in L$ *is meromorphic.*

2°. *For a suitable choice of Blaschke's product*

$$\Pi_+(z) = \prod_{k=1}^{\infty} \frac{z-a_k}{z-\bar{a}_k} \frac{i-\bar{a}_k}{i-a_k} \quad (\mathrm{I}z > 0,\ \mathrm{I}a_k > 0,\ k = 1, 2, \ldots)$$

and positive harmonic function $M_+(z)$ *in the upper half-plane* $(\mathrm{I}z > 0)$ *all functions* $\Pi_+(z) f(z)$, *where* $f(z) \in L$, *are holomorphic for* $\mathrm{I}z = 0$ *and*

$$\log |\Pi_+(z) f(z)| \leqslant M_+(z) + \frac{1}{2} \log B(f, f) - \frac{1}{2} \log y \quad (\mathrm{I}z > 0)$$

3°. *For the lower half-plane exist similar functions* $\Pi_-(z)$ *and* $M_-(z)$.

3. If $B(g, f)$ $(g, f \in L)$ is an H-functional, then by Theorem 1 L_B is a separable space. Let $\{\varphi_k\}_1^{\infty}$ be a certain complete orthonormed basis of the space L_B. If $1 \in L$, then the functions of z

$$\Phi_k(z, \zeta) = \frac{\varphi_k(z) - \varphi_k(\zeta)}{z - \zeta} \quad (\zeta \in G_{\varphi_k},\ k = 1, 2, \ldots)$$

belong to L. Put in this case

$$\psi_k(\zeta) = B(\Phi_k, 1) \quad (\zeta \in G_{\varphi_k},\ k = 1, 2, \ldots)$$

In order to be able to formulate the following theorem, we need

Definition 3. A point a of the complex plane will be called L-regular, if there exists a neighbourhood of this point, in which every function $f(z) \in L$ is holomorphic.

The set of all L-regular points we denote by G_L.

Theorem 4. *If* $1 \in L$ *and the H-functional* $B(g, f)$ $(g, f \in L)$ *possesses more than one distribution function, then*

1°. *The series*

$$\sum_1^{\infty} |\varphi_k(z)|^2, \quad \sum_1^{\infty} |\psi_k(z)|^2$$

converge uniformly on every bounded closed set of L-regular points.

2°. *There exist 4 holomorphic in* G_L *functions* $p_i(z)$, $q_i(z)$ $(i = 0, 1)$ *such that*

$$\left.\begin{aligned} q_0(z)\overline{q_1(\zeta)} - q_1(z)\overline{q_0(\zeta)} &= (z - \bar{\zeta}) \sum_{k=1}^{\infty} \varphi_k(z)\overline{\varphi_k(\zeta)} \\ p_0(z)\overline{p_1(\zeta)} - p_1(z)\overline{p_0(\zeta)} &= (z - \bar{\zeta}) \sum_{k=1}^{\infty} \psi_k(z)\overline{\psi_k(\zeta)} \\ q_0(z)\overline{p_1(\zeta)} - q_1(z)\overline{p_0(\zeta)} &= 1 + (z - \bar{\zeta}) \sum_{k=1}^{\infty} \varphi_k(z)\overline{\psi_k(\zeta)} \end{aligned}\right\} \quad (z, \zeta \in G_L) \qquad (2)$$

3°. *Every distribution function* $\sigma(t)$ *of the functional* B *may be obtained from the relation*

$$\frac{p_0(z) + \tau(z)\, p_1(z)}{q_0(z) + \tau(z)\, q_1(z)} = \int_{-\infty}^{\infty} \frac{d\sigma(t)}{t - z} \quad (\mathrm{I}z > 0) \qquad (3)$$

where $\tau(z)$ $(\mathrm{I}z > 0)$ *is an arbitrary holomorphic function satisfying the condition* $\mathrm{I}\tau(z) \geqslant 0$ $(\mathrm{I}z > 0)$ or $\tau(z) \equiv \infty$.

4°. *Every canonic distribution function* $\sigma(t)$ [*i. e. corresponding to* $\tau(z) \equiv C\ (-\infty < C \leqslant \infty)$] *possesses the property that its derivative vanishes*

almost everywhere. On every interval consisting exclusively of L-regular points, the canonic distribution function $\sigma(t)$ *possesses a finite number of points of increase.*

Observe that the functions $q_i(z)$, $p_i(z)$ $(i=0, 1)$ are determined only up to a linear transformation

$$q_0^* = \omega(\alpha q_0 + \beta q_1) \qquad p_0^* = \omega(\alpha p_0 + \beta p_1)$$
$$q_1^* = \omega(\gamma q_0 + \delta q_1) \qquad p_1^* = \omega(\gamma p_0 + \delta p_1)$$

where $|\omega| = 1$ and α, β, γ, δ are real constants such that $\alpha\delta - \beta\gamma = 1$.

For shortness we omit the general rule of finding the explicit expressions for the functions $q_i(z)$, $p_i(z)$ $(i=0, 1)$ through the functions $\varphi_k(z)$, $\psi_k(z)$ $(k=1, 2, \ldots)$. We remark, however, that if on the real axis there is a L-regular point ξ, we may assume $q_0(\xi) = p_1(\xi) = 1$, $p_0(\xi) = q_1(\xi) = 0$ and hence by (2) for $\zeta = \xi$ we find

$$q_0(z) = 1 + (z-\xi)\sum_1^\infty \overline{\psi_k(\xi)}\,\varphi_k(z), \quad q_1(z) = -(z-\xi)\sum_1^\infty \overline{\varphi_k(\xi)}\,\varphi_k(z)$$

$$p_0(z) = (z-\xi)\sum_1^\infty \overline{\psi_k(\xi)}\,\psi_k(z), \quad p_1(z) = 1-(z-\xi)\sum_1^\infty \overline{\varphi_k(\xi)}\,\psi_k(z).$$

4. In what follows we assume that the intersection of the set G_L with each of the two half-planes (upper and lower) is non-void. Put

$$\Delta(z) = \sup_{f\in L} \frac{|f(z)|^2}{B(f, f)} \quad (z \in G_L) \tag{4}$$

Theorem 5. *In all L-regular points z situated in one and the same half plane (upper or lower) there is simultaneously either* $\Delta(z) = \infty$ *or* $\Delta(z) < \infty$.

Theorem 6. *In order that an H-functional* $B(g, f)$ $(g, f \subset L)$ *should possess only one distribution function, it is necessary and sufficient that at least one half-plane should be* $\Delta(z) = \infty$.

Observe that if at a certain point $z \in G_L$ $\Delta(z) < \infty$, then for this point z

$$\Delta(z) = \sum_{k=1}^\infty |\varphi_k(z)|^2$$

This relation remains valid also at the points $z \in G_L$, where $\Delta(z) = \infty$, if the sequence $\{\varphi_n\}$ is obtained by orthonorming from a sequence $E \subset L$ possessing the property that for every function $f \in L$ and every point $z \in G_L$ there exists a sequence $\{g_n\}$ belonging to the linear envelope of E and such that

$$\lim_{n\to\infty} g_n(z) = f(z), \quad \lim_{n\to\infty} B(f-g_n, f-g_n) = 0$$

In the most interesting cases the aggregates L possess «nuclei» E with the properties indicated. In particular, this is the case where the aggregate L itself is the linear envelope of a certain sequence of functions.

5. By $\mathbf{L}_\sigma^{(2)}$, where $\sigma(t)$ $(-\infty < t < \infty)$ is a non-decreasing function, we shall denote the Hilbert space of all σ-measurable and σ-integrable together with their squares functions $f(t)$ $(-\infty < t < \infty)$ for which the scalar product is defined in the usual way:

$$(g, f) = \int_{-\infty}^{\infty} g(t)\overline{f(t)}\, d\sigma(t) \quad (g, f \in \mathbf{L}_\sigma^{(2)})$$

If $\sigma(t)$ is a distribution function of the functional $B(g, f)$ $(g, f \in L)$, then by (1) the set L of functions may be considered as a part of $\mathbf{L}_\sigma^{(2)}$.

Theorem 7. *If* $1 \in L^*$ *and the H-functional* $B(g, f)$ $(g, f \in L)$ *possesses*

* The condition $1 \in L$ may be omitted if the definition of the canonic function $\sigma(t)$ given in Theorem 3 is appropriately generalized in the case where $1 \in L$.

more than one distribution function $\sigma(t)$, then for those and only those of them which are canonic, L is dense in $L^{(2)}_\sigma$.

Theorem 8. *If the H-functional $B(g, f)$ $(g, f \in L)$ possesses a unique distribution function $\sigma(t)$, then L is dense in $L^{(2)}_\sigma$ in that and only that case where $\Delta(z) = \infty (z \in G_L)$ in both half-planes.*

Denote by $R_\varepsilon (\varepsilon = \pm 1)$ the set of all functions $F(z)$ holomorphic for $\varepsilon \mathrm{I}\, z > 0$ and such that $|F(z)|^2$ possesses in the half-plane $\varepsilon \mathrm{I} z > 0$ a positive harmonic majorant.

Theorem 9. *Let the H-functional $B(g, f)$ $(g, f \in L)$ possess a unique distribution function $\sigma(t)$ and let L be not dense in $L^{(2)}_\sigma$, so that for one of the two values of $\varepsilon (= \pm 1)$:*

$$\Delta(z) = \sum_1^\infty |\varphi_k(z)|^2 < \infty \text{ for } \varepsilon \mathrm{I}\, z > 0, \quad \Delta(z) = \infty \text{ for } \varepsilon \mathrm{I}\, z < 0 \ (z \in G_L)$$

Then:

$$1^\circ.\ (z - \bar{\zeta}) \sum_1^\infty \varphi_k(z) \overline{\varphi_k(\zeta)} = [\Phi(z) \overline{\Phi(\zeta)}]^{-1} \quad (\varepsilon \mathrm{I}\, z > 0,\ \varepsilon \mathrm{I}\, \zeta > 0) \tag{5}$$

where $\Phi(z)$ is the function, holomorphic for $\varepsilon \mathrm{I}\, z > 0$, defined up to a constant factor c, $|c| = 1$.

2°. *Almost for all real λ $\sigma'(\lambda) = |\Phi(\lambda)|^2 / 2\pi > 0$, where $\Phi(\lambda)$ is the boundary value of $\Phi(z)$ existing almost for all real λ on all non-tangent paths.*

3°. *In order that a certain function $f(z)$ $(\varepsilon \mathrm{I}\, z > 0,\ z \in G^L)$ should admit of the decomposition $f(z) = \sum_1^\infty c_k \varphi_k(z)$ where $\sum_1^\infty |c_k|^2 < \infty$ $(\varepsilon \mathrm{I}\, z > 0,\ z \in G_L)$, it is necessary and sufficient that $(z + \varepsilon i)^{-1} \Phi(z) f(z) \in R_\varepsilon$. This condition satisfy all functions $f \in L$.*

4°. *If $1 \in L$, then*

$$-\infty < \int_{-\infty}^{\infty} \frac{\log \sigma'(\lambda)}{1 + \lambda^2}\, d\lambda \quad (< \infty)$$

$$\Phi(z) = \text{const}\ \Pi(z) \exp \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{1 + \lambda z}{\lambda - z} \frac{\log \sigma'(\lambda)}{1 + \lambda^2}\, d\lambda \quad (\varepsilon \mathrm{I}\, z > 0), \tag{6}$$

where $\Pi(z)$ $(\varepsilon \mathrm{I}\, z > 0)$ is the Blaschke product composed of the least number of factors in such a way that all functions $\Pi(z) f(z)$ $(f \in L)$ are holomorphic in the half-plane $\varepsilon \mathrm{I}\, z > 0$.

Choosing for L one or the other aggregate of functions, we obtain from Theorems 1—8 the results of H. Hamburger, M. Riesz and R. Nevanlinna pertaining to the problem of moments ([3]), as well as the complete solution of the problem of Nevanlinna-Pick ([4]) and of the problem of continuation of positive definite functions ([5]). In the following notes we shall give the applications of our theorems to the theory of infinitely decomposible positive-definite functions and to the theory of J. von Neumann — J. Schoenberg ([6]) and A. Kolmogoroff ([7]) of screw-lines in the Hilbert space.

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REFERENCES

[1] M. S. Livshitz, C. R. Acad. Sci. URSS, XLIV, No. 1 (1944). [2] M. Krein, ibid., XLIII, No. 8 (1944); XLIV, No. 4 (1944); XLIV, No. 5 (1944). [3] J. A. Shohat and J. D. Tamarkin, The problem of moments, New York, 1943. [4] R. Nevanlinna, Ann. Ac. Sc. Fenn., serie K, 32 (1929). [5] M. Krein, C. R. Acad. Sci. URSS, XXVI, No. 1 (1940). [6] J. von Neumann and J. Schoenberg, Trans. Amer. Math. Soc., 50 (2) (1941). [7] A. N. Kolmogoroff, C. R. Acad. Sci. URSS, XXVI, No. 1 et 2 (1940).

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ASTRONOMY

ON THE THEORY OF BRIGHTNESS FLUCTUATIONS IN THE MILKY WAY

By V. A. AMBARZUMIAN, Member of the Armenian Academy of Sciences

By now we may look upon it as an established fact that the absorbing matter in our Galaxy is a flocculent structure of individual clouds whose integral effect is the general cosmic absorption. Those of the absorbing clouds near to us which have a high absorbing power are observed as dark nebulae. The various dark nebulae have been dealt with separately in a vast number of papers. But it is no less interesting to investigate statistically the totality of absorbing clouds including those of small absorbing power.

In one of his previous works [1] the author was able to show that fluctuations in the observed number of extragalactic nebulae (Hubble) are due to the flocculency of absorbing matter in the Galaxy. He deduced from these fluctuations a certain mean value of optical thickness for every cloud. It turned out to be $0^m.27$.

As a working hypothesis it may be assumed that the absorbing clouds are also responsible for fluctuations in the apparent distribution of brightness along the Milky Way. The statement appears to constitute a first approximation to the true state of affairs. The question arises, What is the law of brightness distribution if the above fluctuations have really such origin. Below we establish a differential equation which the function of brightness distribution will satisfy and from which the respective moments (mathematical expectations) of all orders may easily be derived.

1. Let us suppose that stars and absorbing clouds fill with uniform density the whole equatorial plane of the Galaxy to infinity. This assumption, though quite arbitrary, is not contrary to reason, since absorption is so strong at great distances that the stars and dark clouds existing there do not practically influence the observed brightnesses. To begin with we shall suppose also that an invariable portion of light $1-q$ is absorbed by a cloud when the light from stars passes through it so that its transparency is q.

If the number of clouds in a certain direction up to distance s from the observer be denoted by $n(s)$ (this function, evidently, assumes chance values), then the light from a star located at the above distance will be weakened $q^{n(s)}$ times.

It follows from this that if an elementary volume of the Galactic space dV radiates in a unit solid angle an amount of energy $\eta\, dV$ (we mean the total radiation of the stars comprised within this element), then the total brightness in a given direction in the plane of the Galactic equator will be

$$\int_0^\infty q^{n(s)} \eta\, ds$$

We are to compute the probabilities for different values of this integral. Denote by $f(I)$ the probability of this integral being larger than I:

$$f(I)=p\left(\int_0^\infty q^{n(s)}\eta\,ds>I\right)$$

It is then obvious that

$$f(I)=p\left(\int_0^a q^{n(s)}\eta\,ds+q^{n(a)}\int_a^\infty q^{n(s)-n(a)}\eta\,ds>I\right) \tag{1}$$

where we have introduced a small positive quantity a.

a being small, we have either $n(a)=0$ or $n(a)=1$. The probability of the former value is $1-\nu a$ and of the second, νa. Here ν is the mean number of clouds within a unit path of the ray. The probabilities of $n(a)$ taking other values are quantities of higher order than a and will therefore be neglected. Correspondingly, $\int_0^\infty q^{n(s)}\eta\,ds$ can assume either the value ηa, or the value $\eta\theta a$ where $0<\theta<1$.

Therefore

$$f(I)=(1-\nu a)\,p\left(\int_a^\infty q^{n(s)-n(a)}\eta\,ds>I-a\eta\right)+\nu a p\left(\int_a^\infty q^{n(s)-n(a)}\eta\,ds>\frac{I-\eta\theta a}{q}\right) \tag{2}$$

But the absorbing clouds being evenly distributed in space, we have

$$p\left(\int_a^\infty q^{n(s)-n(a)}\eta\,ds>I\right)=p\left(\int_0^\infty q^{n(s)}\eta\,ds>I\right)$$

so that equation (2) may be rewritten as follows:

$$f(I)=(1-\nu a)f(I-a\eta)+\nu a f\left(\frac{I-\eta\theta a}{q}\right) \tag{3}$$

or, neglecting terms of the second order with respect to a

$$f(I)=f(I)-\nu a f(I)-a\eta f'(I)+\nu a f\left(\frac{I}{q}\right)$$

whence

$$f(I)+\frac{\eta}{\nu}f'(I)=f\left(\frac{I}{q}\right) \tag{4}$$

Introducing a new variable $u=I\frac{\nu}{\eta}$, we give this equation the form

$$f(u)+f'(u)=f\left(\frac{u}{q}\right) \tag{5}$$

We adopt the following notation for the density of the probability $f'(u)=g(u)$. For it we obtain by differentiating (5) the equation

$$g(u)+g'(u)=\frac{1}{q}g\left(\frac{u}{q}\right) \tag{6}$$

From this functional equation one may easily obtain the mathematical expectation of the brightness u itself, of its square, and so on.

To do so we first realize from (5) that for $u=0$ we have

$$f'(0)=g(0)=0$$

Multiplying (6) by u and integrating over the entire domain $(0, \infty)$ we get

$$\overline{u} + \int_0^\infty g'(u)\, u\, du = q\overline{u}$$

where the dash marks the mean value. Integrating by parts and using the condition of normalization

$$\int_0^\infty g(u)\, du = 1$$

we obtain

$$\overline{u} = \frac{1}{1-q} \tag{7}$$

In a similar way, multiplying (6) by u^2 we shall find upon integration

$$\overline{u^2}(1-q^2) = -\int_0^\infty g'(u)\, u^2\, du = 2\overline{u}$$

whence

$$\overline{u^2} = \frac{2\overline{u}}{1-q^2} = \frac{2}{(1-q)^2(1+q)}$$

For the relative mean square deviation we get

$$\frac{\overline{(u-\overline{u})^2}}{\overline{u^2}} = \frac{\overline{u^2}}{\overline{u^2}} - 1 = \frac{1-q}{1+q} \tag{8}$$

It may be seen that the relative value of the mean square deviation is entirely determined by the transparency value q of a cloud.

2. Now we shall drop the assumption that all absorbing clouds have equal optical thickness. We suppose them in fact to have different optical thicknesses, but at the same time we assume that on the transition from one region of the space to another region the distribution of clouds in respect to the optical thickness rests invariant. Let $d\varphi(q)$ be the probability that a ray of light traversing a cloud will find its transparency to be confined between q and $q+dq$.

We then may again deduce a functional equation for the function of brightness distribution

$$f(u) + f'(u) = \int_0^1 f\left(\frac{u}{q}\right) d\varphi(q) \tag{9}$$

From this we can proceed to the density of probability $g(u)$

$$g(u) + g'(u) = \int_0^1 g\left(\frac{u}{q}\right) \frac{d\varphi(q)}{q} \tag{10}$$

For the mathematical expectations $\overline{u}$ and $\overline{u^2}$ we have

$$\overline{u} = \frac{1}{1 - \int_0^1 q\, d\varphi(q)}, \qquad \overline{u^2} = \frac{2\overline{u}}{1 - \int_0^1 q^2\, d\varphi(q)} \tag{11}$$

Thus, if we form the averages

$$\overline{q} = \int_0^1 q\, d\varphi(q), \qquad \overline{q^2} = \int_0^1 q^2\, d\varphi(q) \tag{12}$$

then

$$\overline{u} = \frac{1}{1-\overline{q}}, \qquad \overline{u^2} = \frac{2\overline{u}}{1-q^2} > \frac{2\overline{u}}{1-\overline{q}^2}$$

because

$$\overline{q^2} > \overline{q}^2$$

Therefore in this case the relative mean square deviation

$$\frac{\overline{u^2} - \overline{u}^2}{\overline{u}^2} = \frac{\int_0^1 (1-q)^2 \, d\varphi(q)}{\int_0^1 (1-q^2) \, d\varphi(q)}$$

turns out to be greater than if all clouds had the same q equal to $\overline{q}$.

There are a number of absorbing cloud transparency distribution laws $\varphi(q)$ for which equation (10) admits a solution in the form of a definite integral. Thus, if $\varphi(q) = q$, *i. e.* if all transparencies are equiprobable

$$g(u) = \int_{-\infty}^{\infty} e^{iut} \frac{dt}{(1+it)^2} \tag{13}$$

If $\varphi(q) = q^2$, *i. e.* if $\overline{q} = {}^2/_3$ (which is a closer approximation to the actual state of things), then

$$g(u) = \int_{-\infty}^{\infty} e^{iut} \frac{dt}{(1+it)^3} \tag{14}$$

The results obtained connect the brightness fluctuations of the Milky Way with the characteristics of the totality of absorbing clouds.

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REFERENCES

[1] В. А. Амбарцумян, Бюллетень Абастуманской астрофизич. обсерватории, 4, 17 (1940).

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MECHANICS

ON TWO NEW LOCI IN THE KINEMATICS OF PLANE MECHANISMS

By I. I. ARTOBOLEVSKY, Corresponding Member of the Academy

In an earlier paper (1) several mechanisms have been discussed by the author by way of example in order to show how simple geometric constructions can be used for determining the velocity of displacement $\overline{u}$ of the instantaneous centre of rotation of a link in a plane mechanism. The method reduces to finding in the velocity diagram the vector representing the velocity of the hinge centre of an added binary link group with one rotary and two translatory kinematic pairs.

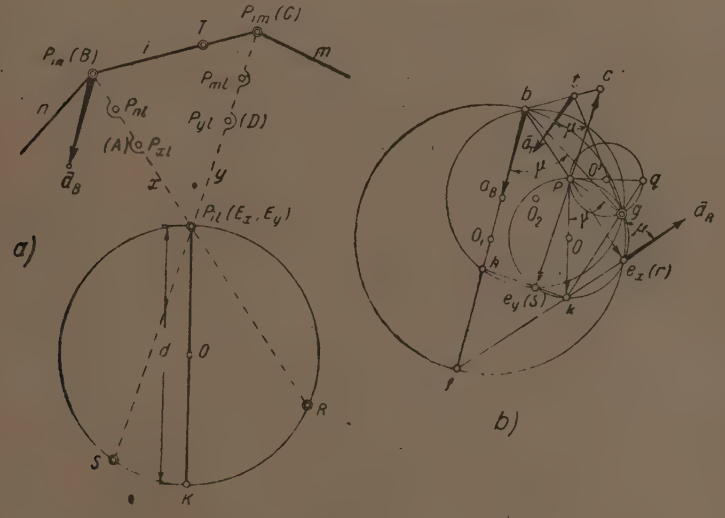

Fig. 1

In the present paper the author gives a general solution of this problem for a link of an arbitrary plane kinematic chain. We are to find the velocity $\overline{u}$ of a link i forming kinematic pairs with links n and m (Fig. 1,*a*). Denote by P_{nl} and P_{ml} the instantaneous centres of rotation of the links n and m relative to a fixed link l; and by P_{in}, P_{im}, the instantaneous centres of rotation of the link i relative to links n and m. The instantaneous centre of rotation P_{il} lies notoriously at the intersection of the straight lines connec-

ting respectively the above centres. The directions connecting these centres can, as pointed out in [1], be conventionally represented as links x and y added to the main kinematic chain, and, as the instantaneous centres P_{xn} and P_{ym} will coincide with P_{in} and P_{im}, respectively, the instantaneous centres P_{xl} and P_{yl} of the links x and y will be found lying on the directions of the axes of these links. The movement of the link i for the instant under consideration can now be represented as the movement of a connecting rod BC conceived as a part of some hinged four-bar mechanism $ABCD$.

We plot a rotationally displaced velocity diagram (pbc) of this mechanism (Fig. 1,b) in the customary way and find in it the points e_x and e_y which mark the ends of the vectors representing the velocities of the points E_x and E_y. The points E_x and E_y belong to the links $AB(x)$ and $CD(y)$ and coincide, while in their present position, with the centre P_{il}. We further draw a circumference with centre o through the points p, e_x and e_y. The segment (pk), a diameter of this circumference, is just the velocity vector $\overline{u}$ plotted to scale and turned through an angle of 90°. Knowing the vector $\overline{u}$, one may easily construct the so-called La Hire's circle [2], *i. e.* the locus of those points of link i which have only tangential accelerations. The diameter d of La Hire's circle is known to be equal to $d = u/\omega_i$ where ω_i is the angular velocity of the link i.

Because $u = \mu_v(pk)$ and $\omega_i = \mu_\omega\left(\frac{bc}{BC}\right)$, where ($pk$), ($bc$) and ($BC$) are the respective segments in the scheme and velocity diagram of the mechanism, μ_v and μ_ω are the plotting scales, therefore

$$d = \frac{\mu_v}{\mu_\omega}\left(\frac{pk}{bc}\right)BC$$

Since the segment (pk) (Fig. 1,b) representing the velocity $\overline{u}$ in the velocity diagram has been turned through an angle of 90°, the diameter d of the rotation circle is parallel to it. If, therefore, we draw through point P_{il} (Fig. 1,a) a straight line parallel to segment (pk) and lay off upon it a segment $P_{il}K$ equal in magnitude to the value of d obtained above, the circumference with centre o having this segment for its diameter will be exactly the circle of La Hire.

Let us further show that in the kinematics of plane mechanisms every problem connected with La Hire's circle can be solved by using the just obtained locus in the velocity diagram, *viz.* the circumference with centre o (Fig. 1, b). This we shall do by proving that this circumference is the locus of ends of vectors representing the turned velocities of those points of the link i which have only tangential accelerations. In other words, the circumference with centre o represents La Hire's circle in the velocity diagram. The points S and R (Fig. 1, a) of the link i belonging to La Hire's circle are bound with points A, B, C, D and with the centre P_{il} by the relations

$$\frac{BP_{il}}{BR} = \frac{AB}{BP_{il}}, \qquad \frac{CP_{il}}{CS} = \frac{DC}{CP_{il}}$$

Because by construction $\frac{AB}{BP_{il}} = \frac{pb}{be_x}$ and $\frac{DC}{CP_{il}} = \frac{pc}{ce_y}$, therefore $\frac{BPil}{BR} = \frac{pb}{be_x}$ and $\frac{CP_{il}}{CS} = \frac{pc}{ce_y}$, in other words points r and s which are velocity diagram images of points R and S, coincide with points e_x and e_y. Then the point k of the circumference o will represent the turning pole K.

A circumference thus constructed can serve to meet all those fundamental problems in the kinematics of mechanisms which are solved by means of Hire's circles. If, for instance, the acceleration vector $\overline{a}_B$ of point B is given, the accelerations of the remaining points of the link can be determined by finding on the constructed circumference the image of the instantaneous centre

of accelerations G. For this purpose we draw through point b (Fig. 1, b) the direction of the vector $\bar{a}_B$ and find the point f at which it meets the direction rk of the acceleration vector $\bar{a}_R$ of the point R. The intercept (bf) thus obtained we take for the diameter of the circumference which we draw from point o_1 as centre. As the angle be_xf is equal to 90°, this circumference will cross that drawn about o at points e_x and g. Indeed, the points b, g, e_x (r) and f lying all on the same circumference with centre o_1, the vectors $\bar{a}_B$ and $\bar{a}_R$ will both form with the radii vectors, connecting point g with points b and $e_x(r)$, the same angle μ, which means that point g is actually the image of the instantaneous centre of accelerations G. The point g can also be found at intersection of the circumference having its centre at o with another circumference drawn about o_2 through points b, $e_y(s)$ and h, where h is the intersection point of vectors $\bar{a}_B$ and $\bar{a}_S$.

The angle μ is obtained by connecting point g with point p. We have

$$\cos \mu = \frac{V_G}{V_K} = \frac{pg}{pk}$$

where V_G and V_K are the velocities of points G and K.

Any point T of the link i (Fig. 1, a) will have its acceleration $\bar{a}_T$ determined, if we find its image point t in the diagram (Fig. 1, b) and, connecting it with point g, draw at g a vector $\bar{a}_T$ to form an angle μ with the direction gt.

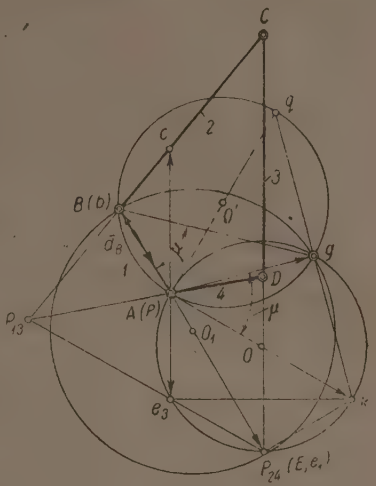

Fig. 2

For the accelerations $\bar{a}_B$, $\bar{a}_C$ and $\bar{a}_T$ *etc.* we shall thus find

$$a_B = \mu_a(gb), \quad a_C = \mu_a(gc), \quad a_T = \mu_a(gt) \text{ etc.}$$

where μ_a is the scale of construction.

The angular acceleration ε_i of the link i will be determined from the condition

$$\varepsilon_i = \omega_i^2 \operatorname{tg} \mu = \mu_s \left(\frac{bc}{BC}\right)^2 \left(\frac{kg}{pg}\right)$$

where μ_s is the construction scale; (bc), (kg), (pg) and BC are the respective segments of the velocity diagram and the scheme; ω_i is the angular velocity of the link i.

It will be obvious that all data on the accelerations of the link are fully determined by the constructed locus.

Another useful locus in the velocity diagram is the circumference whose points are the ends of the velocity vectors of those points of the link which have only normal accelerations. To construct this circumference we draw a straight line from k to g (Fig. 1, b) and produce it until it meets at point q with the perpendicular to pk erected from point p. Then we draw a circumference having the intercept pq for diameter and o' for centre. Angle pqg being equal to μ, any point of this circumference will represent in the velocity diagram a point of link i whose total acceleration passes through the instantaneous centre of rotation; in other words, the constructed circumference gives in the velocity diagram the so-called circumference of Eress [3].

Fig. 2 shows the method of finding the above two loci in the case of a four-bar chain $ABCD$ with fixed link *4*, and with its driving crank *1* rotating at constant angular speed. We find the instantaneous centres of rotation P_{24} and P_{13} at the points where the axes of the links *1*, *3* and *2*, *4* intersect. At

the point $A(p)$ we construct a rotationally displaced velocity diagram pbc to the scale of crank 1. We produce the intercept (pc) until it meets at point e_3 with the direction $P_{13}P_{24}$. The circumference with centre o running through the points p, e_3 and $P_{24}(e_1)$ is the first of the above-mentioned loci.

We, further, find the position of the point g. This is done by drawing from o_1 a circumference which has the intercept (be_1) for its diameter. Its intersection with the circumference drawn from o will give the point g.

For finding the second locus we draw a straight line from point k, the end of diameter pk of the circumference with centre o, to point g and produce it to cross at point q the perpendicular to pk erected from point p. The circumference we are looking for is obtained if we take the intercept pq for its diameter and o' for its centre. By construction, the angles pbg and pqg being equal, this circumference will obviously pass through the point $B(b)$, too. This it should do indeed, because the angular velocity of the crank has been assumed to be constant.

Received
23.II.1944.

REFERENCES

[1] И. И. Артоболевский, Геометрические методы решения некоторых задач теории плоских механизмов, Труды семинара по теории механизмов и машин, 1944. [2] De La Hire, Traité de roulettes, Paris, 1706. [3] Bresse, Journ. de l'école impériale polytechnique, 1853.

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MECHANICS

POISSON'S THEOREM IN NON-HOLONOMIC COORDINATES

By V. V. DOBRONRAVOV

(Communicated by N. E. Kochin, Member of the Academy, 25.I.1944)

In the present paper the classical Poisson's theorem dealing with the properties of integrals of the canonical equations of dynamics* is extended to the case where the latter are represented in non-holonomic coordinates (quasi-coordinates). Further we give the solution of the problem of D. Fobylev (2) and N. Joukovsky (3) of the motion of a ball on a horizontal plane as an example of application of the Jacobi method to the non-holonomic mechanical systems; this is an addendum to the previous papers of the author (4) on the Hamilton-Jacobi theory in non-holonomic coordinates.

Hamilton's canonical equations in non-holonomic coordinates have the form (4)

$$\frac{d\pi_\lambda}{dt} = \frac{\partial H}{\partial p_\lambda}; \qquad \frac{dp_\lambda}{dt} = -\left(\frac{\partial H}{\partial \pi_\lambda}\right) + \sum_{s=1}^{n}\sum_{r=1}^{n} \gamma^s_{r\lambda}\, p_s \frac{\partial H}{\partial p_r} \tag{1}$$

By the derivative with respect to a non-holonomic coordinate we mean the operator

$$\left(\frac{\partial F}{\partial \pi_\lambda}\right) \equiv B_\lambda(F) \equiv \sum_{\rho=1}^{n} \frac{\partial F}{\partial q_\rho}\beta_{\lambda\rho} \tag{2}$$

where $\beta_{\rho\lambda}$ are coefficients of the transformation of holonomic coordinates into non-holonomic ones:

$$dq_\mu = \sum_{\lambda=1}^{n} \beta_{\lambda\mu}\, d\pi_\lambda \tag{3}$$

inverse transformation is

$$d\pi_\lambda = \sum_{\mu=1}^{n} \alpha_{\lambda\mu}\, dq_\mu \tag{4}$$

$\gamma^s_{r\lambda}$ are Ricci-Hamel's coefficients, *i. e.*

$$\gamma^s_{r\lambda} = \sum_{k=1}^{n}\sum_{\mu=1}^{n} \left(\frac{\partial \alpha_{sk}}{\partial q_\mu} - \frac{\partial \alpha_{s\mu}}{\partial q_k}\right)\beta_{rk}\,\beta_{\lambda\mu} \tag{5}$$

* Jacobi(1) proclaimed it to be one of the most remarkable theorems in the integral calculus and the most profound discovery among those made by Poisson.

For any two functions $f_1(q_i, p_i, t)$ and $f_2(q_i, p_i, t)$ we introduce the following expression denoted by double parentheses

$$((f_1, f_2)) \equiv \sum_{\lambda=1}^{n} \left(\frac{\partial f_1}{\partial \pi_\lambda} \frac{\partial f_2}{\partial p_\lambda} - \frac{\partial f_1}{\partial p_\lambda} \frac{\partial f_2}{\partial \pi_\lambda} \right) + \sum_{i=1}^{n} \sum_{s=1}^{n} \sum_{r=1}^{n} \gamma_{ri}^{s} \frac{\partial f_1}{\partial p_r} \frac{\partial f_2}{\partial p_i} p_s \tag{6}$$

(in the notation of the derivatives with respect to non-holonomic coordinates the parentheses are omitted).

These expressions which may be called generalized Poisson brackets possess the properties of the usual Poisson brackets. Namely: $((f_1, f_2)) + ((f_2, f_1)) \equiv 0$, which follows from the properties of Hamel's coefficients

$$\gamma_{ir}^{s} + \gamma_{ri}^{s} \equiv 0 \tag{7}$$

The brackets (6) satisfy also the generalized Jacobi identity:

$$((f_1((f_2, f_3)))) + ((f_2((f_3, f_1)))) + ((f_3((f_1, f_2)))) \equiv 0 \tag{8}$$

To verify this identity we may either apply the considerations similar to those for the case of usual Poisson brackets, or perform a direct calculation based upon (7) and the following properties of the derivatives with respect to the non-holonomic coordinates:

$$\frac{\partial^2 F}{\partial p_i \partial \pi_j} \equiv \frac{\partial^2 F}{\partial \pi_j \partial p_i} \tag{9}$$

$$\frac{\partial^2 F}{\partial \pi_i \partial \pi_j} \equiv \frac{\partial^2 F}{\partial \pi_j \partial \pi_i} + \sum_{\tau=1}^{n} \gamma_{ji}^{\tau} \frac{\partial F}{\partial \pi_\tau} \tag{10}$$

The first property is evident, the second follows from the properties of the infinitesimal operators $B_\lambda(F)$ (S. Lie's operators).

Let, finally, $f(q_i, p_i, t) = \text{const}$ be a first integral of the equations (1). Let us show that f satisfies identically the condition

$$\frac{\partial f}{\partial t} + ((f, H)) \equiv 0 \tag{11}$$

In fact, if we take the complete derivative of f with respect to the time t and replace $\frac{dq_i}{dt}$ and $\frac{dp_i}{dt}$ by their expressions from (3) and (1), we shall have

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{n} \frac{\partial f}{\partial p_i} \left(-\frac{\partial H}{\partial \pi_i} \right) + \sum_{i=1}^{n} \sum_{s=1}^{n} \sum_{r=1}^{n} \gamma_{ri}^{s} \frac{\partial f}{\partial p_r} \frac{\partial H}{\partial p_i} p_s + \\ + \sum_{\lambda=1}^{n} \sum_{\mu=1}^{n} \frac{\partial f}{\partial q_\mu} \beta_{\lambda\mu} \frac{\partial H}{\partial p_\lambda} \equiv 0 \tag{12}$$

Substituting λ for the index i in the first sum in (12) and introducing again the derivatives with respect to the quasi-coordinates according to (2) in the third sum in (12), we shall obtain the identity (11).

The properties of the Poisson brackets just established enable us to prove the following theorem:

If $f_1(p_i, q_i, t) = C_1$ and $f_2(p_i, q_i, t) = C_2$ are any two integrals of the canonical system (1) *in non-holonomic coordinates, then the equality* $((f_1, f_2)) = \text{const}$ *yields, in general, a third integral of this system*.*

For the equations in Poincaré-Lie's variables, *i. e.* for the case of holonomic mechanical systems with $\gamma_{ri}^{s} = \text{const}$, an analogous theorem was stated by N. Četajew (6). Our theorem deals with the most general case where the transfor-

* The proof can be carried out just in the same way as in the case of the Poisson theorem for holonomic coordinates (5).

mations (3) are not supposed to form a transitive group, which is necessary for Poincaré-Lie's variables, *i. e.* the coefficients γ^s_{ri} may depend on q_i and, consequently, on π_i. It is a striking fact that, when we verify the generalized Jacobi identity, the derivatives of Hamel's coefficients with respect to the non-holonomic coefficients vanish identically. At the same time these coefficients cannot depend on the quasi-impulses p_i, being, by their very structure, not dependent on the quasi-velocities.

In particular, it is easy to verify that for the case of scleronomic systems the function H satisfies the identity (11), hence the equations (1) have an integral $H = \text{const}$.

It is necessary to note here that the generalization of the Poisson theorem established above holds for non-holonomic systems too, because, when there are m non-holonomic constraints imposed on the system, the non-holonomic coordinates can be so chosen as to represent the constraints in the following simple form: $d\pi_l/dt = 0$ or $\pi_l = \text{const}$ $(l = 1, 2, \ldots, m)$ (the non-holonomic systems referred to non-holonomic coordinates take on a holonomic form).

As an illustration of the last remark we shall give a solution by Jacobi's method of a problem of non-holonomic motion. Consider a ball of mass M and of radius R, whose central ellipsoid of inertia is an ellipsoid of revolution. Suppose that the motion of the ball consists of rolling and rotation (without sliding) by inertia along a horizontal plane. The geometrical centre coincides with the centre of inertia. We determine the position of the ball by five geometrical (true) coordinates $q_1 = \psi$, $q_2 = \theta$, $q_3 = \varphi$, $q_4 = x$, $q_5 = y$ (the coordinates of the centre and the Euler angles). Introduce the non-holonomic coordinates by the following equalities, where $d\pi_i/dt$ are denoted by ω_i

$$\left.\begin{aligned} &\omega_1 = \frac{d\psi}{dt},\ \omega_2 = \frac{d\theta}{dt},\ \omega_3 = \frac{d\varphi}{dt} \\ &\omega_4 = \frac{dx}{dt} + R\sin\theta\cos\psi\frac{d\varphi}{dt} - R\sin\psi\frac{d\theta}{dt} \\ &\omega_5 = \frac{dy}{dt} + R\sin\theta\sin\psi\frac{d\varphi}{dt} + R\cos\psi\frac{d\theta}{dt} \end{aligned}\right\} \tag{13}$$

The conditions of constraints will be: $\omega_4 = 0$, $\omega_5 = 0$.

The matrix of the inverse transformation will have the form:

$$\|\beta_{ij}\| = \begin{Vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & +R\sin\psi & -R\sin\theta\cos\psi & 1 & 0 \\ 0 & -R\cos\psi & -R\sin\theta\sin\psi & 0 & 1 \end{Vmatrix} \tag{14}$$

In the Hamilton-Jacobi equation in non-holonomic coordinates, *i. e.* in the equation

$$\frac{\partial V}{\partial t} + H\left[\frac{\partial V}{\partial \pi_i}, q_i, t\right] = 0 \tag{15}$$

the derivatives $\partial V/\partial\pi_i$ must be expressed as follows:

$$\left.\begin{aligned} &\frac{\partial V}{\partial \pi_1} \equiv B_1(V) \equiv \frac{\partial V}{\partial \psi}\cdot 1 + \frac{\partial V}{\partial \theta}\cdot 0 + \frac{\partial V}{\partial \varphi}\cdot 0 + \frac{\partial V}{\partial x}\cdot 0 + \frac{\partial V}{\partial y}\cdot 0 \\ &\frac{\partial V}{\partial \pi_2} \equiv B_2(V) \equiv \frac{\partial V}{\partial \psi}\cdot 0 + \frac{\partial V}{\partial \theta}\cdot 1 + \frac{\partial V}{\partial \varphi}\cdot 0 + \frac{\partial V}{\partial x}R\sin\psi - \frac{\partial V}{\partial y}R\cos\psi \\ &\frac{\partial V}{\partial \pi_3} \equiv B_3(V) \equiv \frac{\partial V}{\partial \psi}\cdot 0 + \frac{\partial V}{\partial \theta}\cdot 0 + \frac{\partial V}{\partial \varphi}\cdot 1 - \frac{\partial V}{\partial x}R\sin\theta\cos\psi - \frac{\partial V}{\partial y}R\sin\theta\sin\psi \\ &\frac{\partial V}{\partial \pi_4} \equiv B_4(V) \equiv \frac{\partial V}{\partial \psi}\cdot 0 + \frac{\partial V}{\partial \theta}\cdot 0 + \frac{\partial V}{\partial \varphi}\cdot 0 + \frac{\partial V}{\partial x}\cdot 1 + \frac{\partial V}{\partial y}\cdot 0 \\ &\frac{\partial V}{\partial \pi_5} \equiv B_5(V) \equiv \frac{\partial V}{\partial \psi}\cdot 0 + \frac{\partial V}{\partial \theta}\cdot 0 + \frac{\partial V}{\partial \varphi}\cdot 0 + \frac{\partial V}{\partial x}\cdot 0 + \frac{\partial V}{\partial y}\cdot 1 \end{aligned}\right\} \tag{16}$$

Since x and y are not involved in the expression for the kinetic energy

$$2T = M[(-R\cos\psi\omega_2 - R\sin\theta\sin\psi\omega_3 + \omega_5)^2 + (R\sin\psi\omega_2 - \\ - R\sin\theta\cos\psi\omega_3 + \omega_4)^2] + A(\sin\varphi\sin\theta\omega_1 + \cos\varphi\omega_2)^2 + \\ + B(\cos\varphi\sin\theta\omega_1 - \sin\varphi\omega_2)^2 + C(\cos\theta\omega_1 + \omega_3)^2 \qquad (17)$$

they both are cyclic coordinates in (15). We may therefore seek the complete integral of (15) as a function of ψ, θ, φ only. In other words, we put $\partial V/\partial x = 0$, $\partial V/\partial y = 0$, or $p_4 \equiv 0$, $p_5 \equiv 0$ in the expression for the operators, which means that constructing the quasi-impulses we have to put $\omega_4 = 0$, $\omega_5 = 0$ in the expression for the kinetic energy T in order to meet the conditions implied by non-holonomic relations. We thus obtain the following partial differential equation ($A = B$) (putting $V = W + ht$):

$$\frac{\left(\frac{\partial W}{\partial\theta}\right)^2}{K_3} + \frac{-2C\cos\theta\frac{\partial W}{\partial\varphi}\frac{\partial W}{\partial\psi} + K_1\left(\frac{\partial W}{\partial\varphi}\right)^2 + K_2\left(\frac{\partial W}{\partial\psi}\right)^2}{K_1K_2 - C^2\cos^2\theta} = 2(U-h) \qquad (18)$$

where $K_3 = MR^2 + A$, $K_2 = MR^2\sin^2\theta + C$, $K_1 = A\sin^2\theta + C\cos^2\theta$, $U = -MgR$, $A = B \neq C$ are the principal central moments of inertia.

The complete integral can easily be obtained by Stäckel's method [4] in the form

$$W = a_1\psi + a_2\varphi + \int\sqrt{2K_3\left[U - h - \frac{-2C\cos\theta a_1a_2 + K_1a_2^2 + K_2a_1^2}{2(K_1K_2 - C^2\cos^2\theta)}\right]}\,d\theta + a_3 \qquad (19)$$

There are, further, elliptic quadratures as in [2]. The problem admits a solution for the case $U = U(\theta)$ as well (this is a generalization of a result due to Bobylev, who has used a cumbersome method of Lagrange's multiplyers together with the general theorems of the dynamics of systems).

The above example, as well as the last remark, show the advantage of the method of non-holonomic coordinates, which enables us to apply Jacobi's method to non-holonomic systems and to receive more general results [7] without using the reducing factor of Tchaplyghin [8].

In the following paper we intend to discuss a generalization of Laurent's theorem [9] for the case of non-holonomic coordinates *.

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REFERENCES

[1] C. G. Jacobi, Werke, IV, S. 143; C. R., 11 (1841). [2] D. Bobylev, Rec. math., XVI (1892). [3] Н. Е. Жуковский, Труды Отд. физ. наук Об-ва любит. естествознания, VI (1893). [4] V. V. Dobronravov, C. R. Acad. Sci. URSS, XXII, No. 8 (1939); XLII, No. 6 (1944); XLIII, No. 3 (1944). [5] Н. Н. Бухгольц, Основной курс теоретич. механики, ч. II, 1939, стр. 179. [6] N. Četajew, C. R., 185, 1577 (1927). [7] E. Crescini, Atti dei Lincei, V, 204 (1889). [8] С. А. Чаплыгин, Труды, I, стр. 205, 1933. [9] H. Laurent, Journ. de Math., (2), 17, 422 (1872).

* Correction. In the article by V. V. Dobronravov (C. R. Acad. Sci. URSS, XLII, No. 6, page 251, line 2 above) instead of $\partial\pi_\lambda/\partial t = \ldots$ must be read $dp_\lambda/dt = \ldots$

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PHYSIQUE

SUR LES COURBES D'AIMANTATION LONGITUDINALE D'UN FIL FERROMAGNÉTIQUE PARCOURU PAR UN COURANT CONTINU

Par G. GORÉLIK, X. GORONINA et I. JOUKOVA

(Présenté par L. I. Mandelstam, de l'Académie, le 10. III. 1944)

1. Des considérations élémentaires exposées par l'un de nous (1) (des considérations analogues avaient été émises antérieurement, comme nous l'avons appris depuis, à propos des expériences Fischer—Barnett) permettent de conclure que:

1) Sous l'action d'un champ H_y suffisamment intense la relation entre les composantes H_x, B_x du champ et de l'induction dans une direction perpendiculaire à H_y devient anhystérétique (univoque).

2) H_y croissant, la courbe représentant cette relation s'incline de plus en plus vers l'axe des H_x.

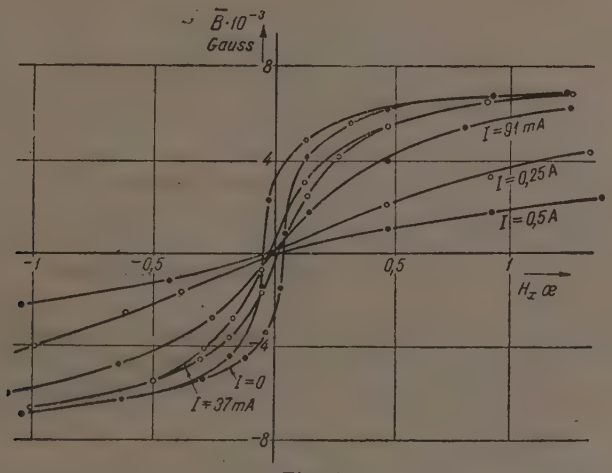

Fig. 1

3) H_y croissant, cette relation devient de plus en plus linéaire.

4) Si H_y est suffisamment grand et H_x suffisamment petit ($H_y > H_s$, $H_x \ll H_s$) on a

$$\mu_x \equiv \frac{B_x}{H_x} = \frac{B_s}{H_y} \qquad \text{(A)}$$

H_s étant la valeur du champ lorsque l'on atteint la saturation dans sa direction (champ de saturation), et B_s l'induction de saturation.

Ces conclusions sont confirmées par l'étude de l'impédance de fils ferromagnétiques rectilignes parcourus par un courant alternatif en fonction du champ magnétique longitudinal, de la fréquence et de l'intensité du courant (1, 2). Dans ces conditions H_y est le champ longitudinal et H_x celui du courant alternatif.

On peut les soumettre à une vérification plus directe en étudiant par la méthode balistique usuelle les courbes d'aimantation longitudinale d'un fil ferromagné-

tique parcouru par un courant continu. H_y est alors le champ du courant, perpendiculaire à l'axe du fil, le champ H_x est axial.

2. Nous avons entrepris cette vérification et obtenu les résultats représentés par les fig. 1 et 2. La fig. 1 se rapporte à un alliage Fe—Ni proche du permalloy avec adjonction de cuivre et de molybdène, de 0.4 mm de diamètre et 40 cm de longueur; la fig. 2 à un fil de permalloy, de meme longueur, de 0.5 mm de diamètre. Des mesures avaient été prises pour que les fils n'aient ni tension, ni torsion. Un petit déplacement des courbes d'aimantation indique que les fils étaient néanmoins légèrement tordus*.

L'examen des fig. 1 et 2 confirme les conclusions qualitatives 1, 2 et 3. Remarquons que lorsque le courant atteint environ 1 A et la courbe d'aimantation longitudinale est pratiquement une droite (jusqu'à des champs longitudinaux de l'ordre de quelques Oe) sa pente est de l'ordre de 10^3 Gauss/Oe, donc encore considérable.

La suppression de l'hystérésis longitudinale d'un fil ferromagnétique sous l'action d'un courant a l t e r n a t i f parcourant le fil est connue depuis longtemps **. Mais à ce que nous sachions, la suppression de l'hystérésis longitudinale par un courant c o n t i n u n'a pas encore été signalée.

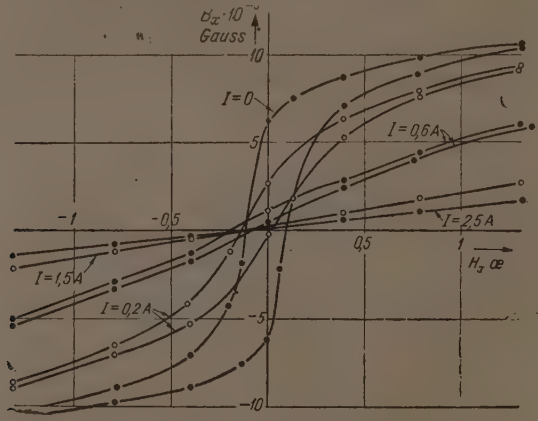

Fig. 2

3. La vérification qualitative de la conclusion 4 nécessite une analyse plus détaillée, étant donné que le champ H_y varie avec la distance à l'axe du fil. L'induction B_x mesurée dans nos expériences est une moyenne:

$$\overline{B_x} = \frac{1}{\pi a^2} \int_0^a B_x(r)\, 2\pi r\, dr = \frac{2}{a^2} \int_0^a B_x(r)\, r\, dr \qquad (1)$$

a étant le rayon du fil. D'après la relation $H_y = 0.2\,\frac{Ir}{a^2}$, $H_a = 0.2\,\frac{I}{a}$ (I — intensité en A, H_a — champ à la surface du fil en Oe) on a

$$\overline{B_x} = \frac{2}{H_a^2} \int_0^{H_a} B_x H_y\, dH_y.$$

---

* L'aimantation longitudinale apparaissant sous l'action conjuguée de la torsion et du courant continu longitudinal a été découverte par Wiedemann [2]. V. également le célèbre Traité de Maxwell [3] et la bibliographie que donne Winkelmann [4] (cet auteur cite également un grand nombre de travaux anciens relatifs à l'action simultanée d'un champ circulaire et d'un champ axial). Augmentant l'angle de torsion de nos fils on peut facilement accentuer le phénomène Wiedemann et le déplacement de la courbe d'aimantation dont il est une des manifestations.

** V. les ouvrages de Spooner [5] et de Wwedenski-Landsberg [6], citant les mémoires originaux [7–12], ainsi que [13–14]. Les conditions actuelles ne nous ont permis de prendre connaissance de tous les mémoires originaux.

Le cas qui nous intéresse est celui où $H_a \gg H_s$. Il est commode d'écrire

$$\overline{B}_x = \frac{2}{H_a^2} \int_0^{H_s} B_x H_y dH_y + \frac{2}{H_a^2} \int_{H_s}^{H_a} B_x H_y dH_y.$$

Si la conclusion 4 est exacte, on peut introduire dans la seconde intégrale l'expression (A), ce qui donne

$$\frac{2B_s H_x}{H_a^2} \int_{H_s}^{H_a} dH_y = \frac{2B_s H_x}{H_a^2} (H_a - H_s)$$

Dans le cas qui nous intéresse ($H_a \gg H_s$) la première intégrale est beaucoup plus petite que la seconde. Pour une évaluation grossière de la première intégrale admettons que μ_x varie selon une loi linéaire dans l'intervalle $0 \leqslant H_y \leqslant H_s$, la valeur $\mu_x = \mu_0$ correspondant à $H_y = 0$ étant donnée par la courbe d'aimantation obtenue en l'absence de courant dans le fil. La première intégrale devient alors $\frac{2H_x}{H_a^2} \int_0^{H_s} \left(\mu_0 - \frac{\mu_0 - \mu_s}{H_s} H_y\right) dH_y$, où $\mu_s = \frac{B_s}{H_s}$ et, par conséquent,

$$\overline{\mu}_x \equiv \frac{\overline{B}_x}{H_x} = \frac{H_s^2}{3H_a^2} (\mu_0 + 2\mu_s) + \frac{2B_s}{H_a^2} (H_a - H_s). \qquad (2)$$

Appliquons cette formule au fil de permalloy pour $I = 2.5$A,

$$H_a = 20 \text{ Oe} \qquad (3)$$

valeur qui dépasse de 10 fois

$$H_s = 2 \text{ Oe} \qquad (4)$$

La fig. 2 donne pour $H_x = 1.32$ Oe la perméabilité

$$\mu_0 = 8\,000 \qquad (5)$$

La valeur mesurée de l'induction de saturation est

$$B_s = 12\,500 \text{ Gauss} \qquad (6)$$

et, par conséquent,

$$\mu_s = 6\,300 \qquad (7)$$

Introduisant (3)—(7) dans (2) nous obtenons $\overline{\mu}_x = 1\,200$ Gauss/Oe.
La courbe expérimentale de la fig. 2 qui correspond à $I = 2.5$ A donne

$$\overline{\mu}_x = 1\,260 \text{ Gauss/Oe}$$

Les valeurs calculées et mesurées de $\overline{\mu}_x$ sont en accord satisfaisant. La conclusion quantitative 4 est donc confirmée.

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Gorki.

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LITTÉRATURE CITÉE

[1] G. Gorélik, C.R. Acad. Sci. URSS, XLIII, No. 5 (1944). [2] G. Wiedemann, Berl. Mon.-Ber. (1860); Pogg. Ann., **117**, 203 (1862). [3] T. C. Maxwell, Traité d'Electricité et de Magnétisme, **2**, Paris, 1889. [4] A. Winkelmann, Handb. d. Physik, **5**, Leipzig, 1908. [5] Th. Spooner, Properties and Testing of Magnetic Materials, 1927. [6] Б. Введенский и Г. Ландсберг, Современное учение о магнетизме, М.—Л., 1929. [7] W. R. Wright, Phys. Rev., p. 161, March (1918). [8] J. R. Ashworth, Phil. Mag., p. 357, February (1914). [9] Steinhaus and Gumlich, Ber. d. Deutsch. Phys. Ges., **17**, 369 (1915). [10] Yasujira Niwa and Yoshihiro Asami, Researches of the Electrotechnical Laboratory, 124, June, 1923. Dep. of Communication, Tokyo, Japan. [11] Gerosa e Finzi, Rend. del Inst. Lomb., **21** (1891). [12] Finzi, Electrician, **26**, 672 (1891). [13] Th. Spooner, Phys. Rev., **25**, 527 (1925). [14] Y. Niwa, T. Matura and T. Sugiura, Researches of the Electrotechnical Laboratory, Dep. of Communications, Tokyo, Japan.

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TECHNICAL PHYSICS

SHUTTER BEFORE AN OBJECT LENS ALWAYS WORKING AT FULL APERTURE

By G. A. TIKHOV, Corresponding Member of the Academy

All the existing shutters mounted in front of the lens open from zero to full aperture, after which they shut the aperture down to zero again. If the total period of opening and shutting makes up but a negligible fraction of the exposure, the above property of the shutter is of no practical importance. If, however, the duration of opening and shutting is comparable with that of the exposure, which may be the case in photographing the Sun, then the property in question is essentially harmful, resulting in a distortion of the image due to diffraction.

In photographing the Sun another type of shutter is also used, *viz.* a more or less narrow slit moving in front of the lens. The slit shutter is also vicious theoretically, since at any given moment only a narrow fraction of the object lens is at work; the narrower this strip, the more the image is impaired by diffraction.

None of the shutters actually applied can thus stand criticism. Theoretically, a slit shutter put in front of the plate is preferable, but it does not hold up in Sun photography, either, since a constant speed of the slit throughout the Sun image cannot possibly be ensured to a necessary degree of accuracy. If even a constant speed were attained, the sufficient accuracy in the slit width all along its length would still remain questionable.

Moreover, in the case of a very narrow slit, diffraction effects and even dust particles on the slit edges may tell upon the image again. Lastly, the different parts of the phenomenon to be photographed would act upon the plate not at the same moment, which, of course, cannot be tolerated in the case of recording air-waves produced by projectiles, *etc.*

Thus, when photographing the Sun and in other cases where the exposure measures some thousandth or hundredth fractions of a second, such a shutter should be aimed at as would always leave open the full aperture of the objective, guaranteing besides an absolute simultaneousness throughout the field to be photographed.

By the end of 1943 I came to the idea that such a shutter is practically possible. The idea was essentially this. Instead of a change in the effective lens area, recourse should be had to a change in its transparency, from full opacity to a certain degree of transparency, and in such a way that the transparency be equal at any point of the lens.

This can be attained with the aid of two photometrical wedges as neutral as possible; the optical density of these should vary linearly with the distance from one of the edges (either the most transparent, or the most opaque). If two such wedges are put together in such a way as to make the transparent end of the one coincide with the opaque end of another, a plate will be obtained with a density uniform throughout its area. Now, if we shift one wedge along another, at the area of their superposition the total density will vary continuously remaining equal

at any given point. If the most opaque part of the wedge is superimposed upon an equal part of another wedge, the resultant density will be a maximum one, and *vice versa*. It remains to mount the wedges into frames which, with the help of a rather simple mechanism, can be made to move in opposite directions, and to gain their original position, corresponding to the maximum opacity. The frames bearing the wedges should be mounted in front of the lens in such a way as to make the latter face the superimposed part of the two wedges.

As the resulting density of the two wedges cannot be increased infinitely (*i. e.* up to absolute opacity), the darkest parts of the wedges should be covered with opaque hemidiscs which will screen the lens when the densest parts of the wedges are superimposed. The theoretical consistency of the design is certainly impaired by introducing these hemidiscs, but this is of no practical importance, for the lens is working at a reduced aperture only at the initial and final small fractions of the exposure, corresponding to the almost opaque superimposed parts of the two wedges.

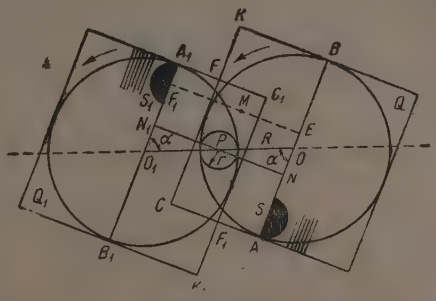

Fig. 1

Mechanically, the drawback of the shutter described consists in that each of the two wedges has to be moved, first in the one direction, and then in the opposite one. In order to avoid this, the following design can be suggested. Imagine (Fig. 1) two identical wedges Q and Q_1 with a linear variation of optical density. Let these wedges be placed into different but parallel planes, and allow them rotate around the axes O and O_1 passing through their centres; let, moreover, the two axes be coupled in such a way as to make the angular velocities of the two wedges equal. Suppose the density of the wedge Q to decrease in the direction AB, while that of the wedge Q_1 decreases in the direction A_1B_1; in the figure this is illustrated by combs with gradually thinning out teeth. This system is mounted in front of the object lens P whose centre is in the middle of the line OO_1.

Suppose that at the initial moment (when the shutter is closed) the points A and A_1 of the wedges are projected upon the straight line OO_1. The densest area of the system of the two wedges will then lie in front of the lens P. Let now the wedges rotate through an angle α. The parts of the wedges projected upon each other will then make up a rectangle CFC_1F_1, where the total density is equal everywhere, since a rotation of the wedges is equivalent to their mutual shift parallel to AB or A_1B_1 to a length FK. Let us find the resultant density at a given point M of the quadrangle CFC_1F_1. The density D of the wedge Q at the point M is proportional to its distance from some zero point a situated somewhere beyond the straight line BK. Denoting the constant of the wedges by k we thus find

$$D = k(BE + a)$$

Similarly, density D_1 of the wedge Q_1 at a point M may be expressed:

$$D_1 = k(B_1E_1 + a)$$

The resultant density will be:

$$D + D_1 = k(BE + B_1E_1 + 2a)$$

But $BE + B_1E_1$ is the distance between the straight lines BK and B_1K_1, and this sum is therefore equal to any point of the quadrangle CFC_1F_1; consequently, the sum of densities within this quadrangle is constant.

It remains to find the distance x between the straight lines BK and B_1K_1. From the diagram it is evident that

$$x = BN + N_1B_1 = 2BN = 2(BO + ON)$$

Denote OP by R and the radius of the lens by r. $BE + B_1E_1$ will then be equal to

$$x = 2(R + r + R\cos\alpha)$$

and

$$D + D_1 = 2k(R + r + R\cos\alpha + a)$$

With $\alpha = 0$ we have: $D + D_1 = 2k(2R + r + a)$, and with α equal to 180° we have $D + D_1 = 2k(r + a)$. Square wedges may be replaced by circular ones having radii $OA = O_1A_1$. In order that the shutter operate at full efficiency, each wedge should be rotated through 360°.

Finally, a third method can be suggested ensuring a non-reduced aperture of the lens. Imagine (Fig. 2) a film $KLMN$ rolled on two cylinders A and B, and consisting of four identical sections: KL, ML, KN and MN.

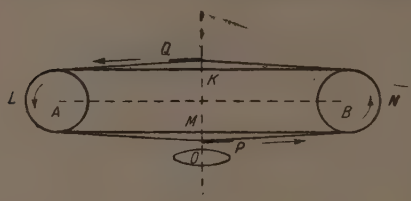

Fig. 2

Each of these sections represents a linear photometrical wedge, the maximum density of which is at the points K and M, and the minimum, at the points L and N. The increase in density is represented in the diagram conventionally by a thickening of the film towards the points K and M. The lens is at O. The breadth of the film somewhat exceeds the diameter of the lens.

At the position shown in the diagram, the densest sections of the film face the lens. To attain absolute opacity, opaque rectangles P and Q are fixed on the film.

Suppose now that after being released the cylinders start rotating in the direction indicated by arrows. The screens P and Q will then be removed from the lens, and the latter will begin to clear up until the points L and N of the film will get in front of it; after this the lens will begin to be shaded until the screens P and Q exchange places.

Theoretically, this system is inferior to the two others, since at the moment of maximum clearing, the lens is cleared not quite uniformly. The shutters here described have, besides, the valuable feature of permitting of an arbitrary increase in exposure, by limiting their swing. Instead of changing the exposure, one may change the swing. The theoretical drawback of the shutter of the third system, which has been mentioned above, can also be eliminated in this way. The reduction in the swing results in that the shutters of the second and third system should, like those of the first system, move first towards the one side, and then towards the opposite one.

Along with all the theoretical advantages of the shutters here described, they have also considerable drawbacks: 1) they are cumbersome; 2) the surfaces of the wedges must be optically polished, for long-focus lenses, at least, and 3) the film of the shutter of the third kind should be enclosed between optically polished glasses at its middle facing the lens.

The theoretical advantages of these shutters are so important, however, that their manufacturing for certain instruments will certainly pay.

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PHYSICAL CHEMISTRY

ON TYPES OF KINETIC CURVES IN CHAIN REACTIONS

III. ALLOWANCE FOR CHAIN RUPTURE ON WALLS OF REACTION VESSEL IN THE CASE OF OXIDATION OF HYDROGEN

By N. N. SEMENOV, Member of the Academy

In the preceding papers [1, 2] we disregarded chain rupture on the walls and in the volume taking place at a velocity $a_1' n = gn$. But we can no longer do so when the reaction is proceeding at pressures comparatively close to the lower limit. Even at $f \gg g$ it leads to inaccuracies in the description of the final stages of the transformation, when $f(1-\eta)$ becomes comparable with g.

Let us examine the influence of chain rupture on the kinetics of the simplest chain reactions using as an example the oxidation of hydrogen at low pressures when we may confine ourselves to chain rupture on the walls alone. If the probability ε of chain rupture upon the collision of an active centre with the wall is extremely small (as is the case in glass vessels), we may consider $a_1' = g$ a constant independent either of the degree of oxidation or of the initial pressure of the mixture [3].

The reaction of the oxidation of hydrogen in the neighbourhood of the lower limit of ignition was experimentally investigated by Kovalsky [4], and a mechanism for this process was advanced by Lewis and Elbe [5].

0) $H_2 + O_2 = 2OH$ — initiation of chains, velocity $n_0 = k_0 (H_2)(O_2)$
1) $H + O_2 = OH + O - 16$ Cal., velocity $a_1 (H) = k_1 (O_2)(H)$ } development
2) $OH + H_2 = H_2O + H + 12$ Cal. » $a_2 (OH) = k_2 (H_2)(OH)$ } of a branched
3) $O + H_2 = OH + H + 0.5$ Cal. » $a_3 (O) = k_3 (H_2)(O)$ } chain
1′) $H + \text{wall} \rightarrow {}^1/_2 H_2 + \frac{101}{2}$ Cal. » $a_1' (H) = g (H)$, chain rupture

Inasmuch as reaction 1, being strongly endothermic, proceeds considerably slower than 2 and 3, then, in accordance with the rules I have given [6], we may assume the derivatives of the concentrations (OH) and (O) with respect to time to be equal to zero, and with the aid of two algebraic equations thus obtained we may express (OH) and (O) through (H) (the number of H atoms in a unit of volume). In this case the problem reduces to integration of the two equations

$$\frac{dn}{dt} = 2n_0 + (2a_1 - a_1')n; \qquad -\frac{d(O_2)}{dt} = n_0 + an \tag{1}$$

where n stands for (H). Introducing the variables used in the previous communications: $\xi = \frac{(H)}{(O_2)_0}$; $\eta = \frac{(O_2)_0 - (O_2)}{(O_2)_0}$; $\tau = ft = k_1 (O_2)_0 t$ and the notations

$n_0 = n_0'(1-\eta)$, $a_1 = f(1-\eta)$ and $\nu_0 = \frac{n_0'}{f(O_2)_0} = \frac{n_0'}{k_1 (O_2)_0^2}$, we may rewrite equations (1) in the form:

$$\frac{d\xi}{d\tau} = 2(\nu_0 + \xi)(1-\eta) - \frac{a_1'}{f}\xi, \quad \frac{d\eta}{d\tau} = (\nu_0 + \xi)(1-\eta) \tag{2}$$

The lower limit of ignition is determined by the condition $2a_1 - a_1' = 0$ when $\eta = 0$. Hence $2k_1 (O_2)_0^x - g = 0$, where $(O_2)_0^x$ is the critical concentration of oxygen at the lower limit, from which it follows that

$$g = 2k_1 (O_2)_0^x \tag{3}$$

and

$$\frac{a_1'}{f} = \frac{g}{k_1 (O_2)_0} = 2\frac{(O_2)_0^x}{(O_2)_0} = \frac{2p_1}{p_0} = R \tag{4}$$

where p_1 is the lower limit of self-ignition (a quantity which can be easily determined experimentally), and p_0 is the initial pressure of the mixture.

In what follows we confine ourselves for simplicity to a consideration of a stoicheiometric mixture.

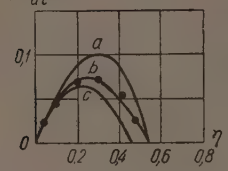

Fig. 1

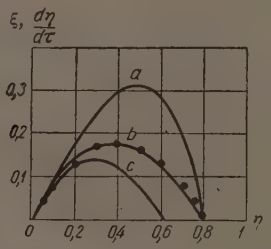

Fig. 2

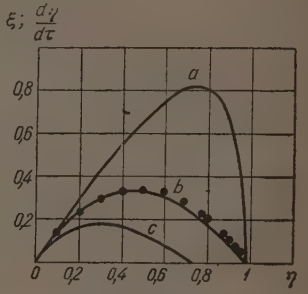

Fig. 3

Let us note that when $\nu_0 = 10^{-7}$ (and precisely such a value of ν_0 follows from the data of Kovalsky), we may discard it in equations (2) for values of ξ and $\eta > 10^{-5}$ and employ the equations in the form:

$$\frac{d\xi}{d\tau} = [2(1-\eta) - R]\xi; \quad \frac{d\eta}{d\tau} = \xi(1-\eta) \tag{5}$$

From these equations we get:

$$\xi - \xi_1 = 2(\eta - \eta_1) + R \ln \frac{1-\eta}{1-\eta_1}$$

Choosing $\xi_1 \sim \eta_1 \sim 10^{-5}$, we may assume $\eta_1 \sim \xi_1 \simeq 0$, for all values of η that are of practical interest, and to express ξ through η the formula

$$\xi = 2\eta + 2.3R \lg(1-\eta) \tag{6}$$

can be used. Curves a in Figs. 1, 2, and 3 show how the concentrations of atomic hydrogen computed with formula (6) for $R = 1.4$ or $p_0 = 1.43\, p_1$ (Fig. 1); $R = 1$ or $p_0 = 2p_1$ (Fig. 2); $R = 0.5$ or $p_0 = 4p_1$ (Fig. 3), change in the course of the reaction.

Curves b in Figs. 1, 2 and 3 show how the velocity with which oxygen is used up or (what amounts to the same thing) half of the velocity of formation of water vapour changes with the extent of transformation η:

$$\frac{d\eta}{d\tau} = \xi(1-\eta).$$

We note the following characteristic properties of the a and b curves.

1. The concentration of hydrogen atoms reaches very big values in the course of the reaction. For $R = 1.4$; 1; 0.5, the partial pressure of atomic hydrogen reaches 10, 30, 80 per cent, respectively, of the partial pressure of the initial oxygen or (for a stoicheiometric mixture) 3.3, 10, 27 per cent, respectively, of the initial pressure of the mixture. Thus, 2.5, 7.5, 20 per cent of the initial molecular hydrogen must be dissociated into atoms in the course of the reaction. Kondratjew (7) and Avramenko diclosed the presence of significant concentrations (tenths of a per cent) of free hydroxyl, formed in the course of the reaction. Inasmuch as at $T = 800°C$ (H) : (OH) $= k_2 : k_1 = 3 \cdot 10^{-13} : 10^{-14}$ (k_2 and k_1 taken from the data of Kovalsky and Kondratjew), for (OH) $\simeq 3 \cdot 10^{-3}$, (H) $\simeq 10$ per cent, which is in qualitative agreement with theory. It would be highly interesting to confirm by direct measurements of (H) the astonishing theoretical conclusion regarding the huge concentrations of atomic hydrogen built up in the course of the reaction of the oxidation of hydrogen at low pressures close to the lower limit.

2. From the equation $\frac{d\xi}{d\eta} = 0$ we find that ξ reaches a maximum at $1 - \eta_m = \frac{R}{2} = \frac{p_1}{p_0} = \frac{(O_2)^x_0}{(O_2)_0}$ or at $(O_2) = (O_2)_0\ (1 - \eta_m) = (O_2)^x_0$. And this is understandable; just as soon as the concentration of oxygen falls in the course of the transformation to values equal to and less than $(O_2)^x_0$, development of the chain avalanche ceases and the reaction continues only at the expense of the atomic hydrogen present, which (as we saw) accumulates by this time in large quantities. When it has all been used up, the reaction stops without going to the end. The residual pressure of the initial mixture $p_1' = p_0(1 - \eta_1') < p_1$ is determined by the equation

$$0 = \xi = 2\eta_1' + R \ln (1 - \eta_1') \qquad (7)$$

Kovalsky has shown experimentally that p_1 is less than p_1', and the smaller it is, the larger p_0. However, the theory gives a somewhat smaller value of p_1' than that obtained by Kovalsky. It is very important to repeat the experiments again very carefully.

3. The curves b in Figs. 1, 2 and 3, which give the velocity $\frac{d\eta}{d\tau}$ as a function of η turn out to be very similar to the autocatalytic curves $w = c\lambda(1 - \lambda)$, where $\lambda = \eta \frac{p_0}{p_0 - p_1'} = \frac{\eta}{\eta_1'}$ (λ is the fraction that has reacted at a given moment τ referred not to the total quantity of the initial mixture, but only to that part of it that undergoes the oxidation). Choosing c equal to 1.37, 0.7 and 0.285 and computing $w = c\lambda(1 - \lambda)$, we find points (on Figs. 1, 2 and 3) that apparently lie very close to the theoretical curves b. From this it follows that Kovalsky's formula $w = cx(p_0 - p_1' - x)$, where x is the quantity of the substance that has already reacted, should hold as an approximation.

However, here it should be noted that the usual conception that the fall in pressure Δp is determined totally by the expenditure of oxygen, is not correct, inasmuch as such tremendous concentrations of atomic hydrogen appear in the course of the reaction. And the kinetics of the oxidation of hydrogen is studied by means of this very fall in pressure (and not the formation of water and the expenditure of oxygen which are very difficult to measure). The actual fall in pressure Δp is proportional to the decrease in the number of molecules Δ(M) in the course of the reaction:

$$\Delta(M) = \Delta(O_2) + \Delta(H_2) - (H_2O) - (H).$$

According to the law of the conservation of matter, $\Delta(O_2) = \frac{(H_2O)}{2}$ and $\Delta(H_2) = (H_2O) + \frac{(H)}{2} = 2\Delta(O_2) + \frac{(H)}{2}$; $\Delta(M) = \Delta(O_2) - \frac{(H)}{2}$ or

$$\pi = \frac{\Delta p}{p_0/3} = \frac{\Delta(M)}{(O_2)_0} = \eta - \frac{\xi}{2} \qquad (8)$$

where π is the fall in pressure Δp (at a given moment), referred to that fall in pressure $p_0/3$ that would have taken place if the entire initial mixture were to react to the end.

Dividing by two the right and left-hand sides of the first of equations (2) and subtracting them respectively from the second equation, we get

$$\frac{d(\eta-\xi/2)}{d\tau}=\frac{d\pi}{d\tau}=\frac{a_1'}{2f}\xi=\frac{R}{2}\xi \qquad (9)$$

The theoretical curves $\frac{d\pi}{d\tau}\left(\frac{d\pi}{d\tau}\right.$ is proportional to $\frac{1}{p_0^3}\frac{d\Delta p}{d\tau}$, where Δp is a quantity measured experimentally as the reaction proceeds) as a function of π may be easily constructed. To do this, by taking one or another value of η we construct with formulae (6), (8) and (9) $\frac{d\pi}{d\tau}$ and π. In Fig. 4 are given such curves for $R=1.4$, 1 and 0.5. For large values of R these curves closely resemble those for autocatalytic reactions; for smaller values of R, however, they are observed to lose their symmetry. For $R=1.43$, $\left(\frac{d\pi}{dt}\right)_{\max}$ is located at 50 per cent transformation; for $R=1$, at 45 per cent; and for $R=0.5$, at 40 per cent. The experimental curves obtained by Kovalsky (for $R>1$, see

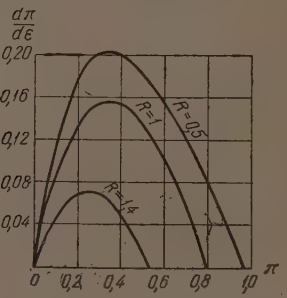

Fig. 4

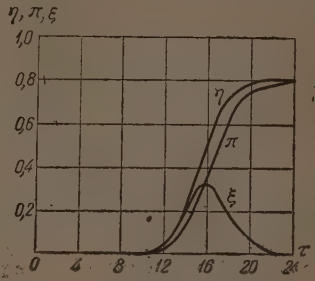

Fig. 5

page 332 of my book «Chain Reactions») lie close to the symmetrical ones, but nevertheless reveal a small shift in the maximum in the direction of small η in compliance with the given theory.

Calculating theoretically $\left(\frac{d(\Delta p)}{dt}\right)_{\max}\sim p_0 p_1 \xi_{\max}$, we may establish that $p_0 p_1 \xi_{\max}$ is approximately proportional to $(p_0-p_1')^2$. It is this that explains the proportionality of the maximum velocity of the reaction to the value $(p_0-p_1')^2$ obtained by Kovalsky experimentally.

Employing the same procedures that were used in the preceding communications, we may plot the quantities ξ, η and π as functions of the time. Corresponding theoretical curves have been plotted in Fig. 5 for $R=1$ and $\nu_0=10^{-7}$.

If chain rupture had not taken place ($g=0$), the reaction would have proceeded stoicheiometrically, according to the following equation:

$$3H_2+O_2+H=2H+2H_2O+H+15 \text{ Cal.}$$

Hence: 1) The reaction would have proceeded without a change in pressure. 2) Its thermal effect would have amounted to 15 Cal. instead of to $2\times 58=116$ Cal.

Only as a result of adsorption of hydrogen atoms by the wall with subsequent recombination of the atoms into molecules of H_2 ($H+H=H_2+101$ Cal.) does a fall in pressure arise, and the full heat of reaction is liberated.

Thus, at low pressures theory leads to the two following fundamental conclusions:

1. The fall in pressure and liberation of the full heat of reaction (58 Cal. per mole of H_2O) should lag behind the expenditure of oxygen and the increase in H_2O.

2. The bulk of the heat of reaction should be given off on the surface of the vessel walls, although the reaction of formation of water proceeds homogeneously in the volume.

Bolometric and manometric measurements of the reaction of the hydrogen — oxygen mixture at low pressures, made by Byron five years ago in the Institute of Chemical Physics, showed that the figure, representing the extent to which the gas heats up during reaction, is several times smaller than that calculated from the rection velocity and total heat of reaction. This seemed to be a paradox and the work was not published. In the light of the theory that has been presented, the results of Byron's work are quite natural.

b'	η_{max}	ξ_{max}
10	0.17	0.06
3	0.29	0.15
1	0.4	0.2
0.5	0.44	0.26
0	0.5	0.31

In conclusion let us analyse the question of the possible rôle of the recombination of the H atoms in the volume ($H+H+M=H_2+M$). We may take this reaction into account by the same method that we employed in communication [2]. As a result, an additional member $b'\xi^2$ enters into the first of the two equations (5) and it is written in the following form:

$$\frac{d\xi}{d\tau}=[2(1-\eta)-R]\,\xi-b'\xi^2 \quad \text{and} \quad \frac{d\eta}{d\tau}=\xi(1-\eta) \tag{10}$$

whence

$$\xi=\frac{1}{b'-1}[1-(1-\eta)^{b'-1}](1-\eta)-\frac{R}{b'}[1-(1-\eta)^{b'}] \tag{11}$$

Here $b'=\frac{x\,(M)}{k_1}$, where x is a constant of the triple collision involved in the reaction $H+H+M=H_2+M$. As is known, $x\simeq 3\cdot 10^{-32}$ and is practically independent of the temperature. According to the experimental data of Kovalsky, k_1 at 800°C is of the order of 10^{-14}, and (M) at pressures of several mm of Hg is of the order of 10^{17}, from which it follows that $b'\sim 0.3-0.5$.

Let us compile a table of the values of ξ_{max} and those values of $\eta=\eta_{max}$ at which ξ_{max} is attained for $R=1$ and various values of b'.

From the table we see that for $R=1$ and $b'=0.5$ the influence of recombination is very slight. However, upon increasing the initial pressure, *i. e.* upon decreasing R, b' increases proportionally to p_0, as a result of which the rôle of recombination grows with a rise in pressure. For this reason the partial pressure of H no longer increases with an increase in p_0. Thus, we should observe huge relative concentrations of H only at low pressures in the neighbourhood of the lower limit of ignition. To obtain an idea about the influence of recombination we give in Figs. 1, 2 and 3 curves of ξ calculated with formula (11) for values of b' that are somewhat too high ($b'=2$ for $R=1.4$; $b'=3$ for $R=1$ and $b'=6$ for $R=0.5$) (Figs. 1, 2 and 3).

From these curves we see that when $p_0=4p_1$, the concentration of H falls from 80 per cent (for $b'=0$) to 18 per cent (for $b'=6$), and the velocity decreases correspondingly. At the same time the residual pressure increases (for $R=0.5$ and $b'=6$, $\eta_1=0.75$ instead of 0.98 for $b'=0$).

Received
7. IV. 1944.

REFERENCES

[1] N. N. Semenov, C. R. Acad. Sci. URSS, XLIII, No. 8 (1944). [2] N. N. Semenov, ibid., XLIV, No. 2 (1944). [3] N. N. Semenof, Acta Physicochim. URSS, XVIII, 93 (1943). [4] Н. Н. Семенов, Цепные реакции, 1934, стр. 290, 332. [5] B. Lewis and G. Elbe, Combustion, Flames and Explosions of Gases, Cambridge, 1938. [6] Н. Н. Семенов, ЖФХ, XVII, 187 (1943). [7] В. Н. Кондратьев, Спектроскопич. изучение газовых химич. реакций, изд-во АН СССР, 1943.

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BIOCHEMISTRY

RESPIRATION OF UNRIPE GRAIN AND ITS STORAGE

By A. I. SMIRNOV, Corresponding Member of the Academy

Insufficiently ripe grain is much more liable to be destroyed during storage than fully mature one. The two main factors accounting for a safe storage of grain are usually believed to be the moisture content of the grain itself, air humidity of the granary, and temperature (1). Mention has been made of the fact that in the case of elevated temperature the grain to be stored soon loses its viability, even if the moisture content is low.

In so far as under identical moisture conditions the respiration of the grain grows more intense according as temperature rises (2), the latter condition is undoubtedly the most important for a safe storage of grain. In fact, along with a rise of temperature, there is an increased evolution not only of carbon dioxide which substitutes for oxygen in the intergranular spaces of the bulk of grain to be stored, but also of water which saturates the air of the intergranular spaces. It is the latter circumstance which affects badly the preservation of grain.

From what has been said above it is but natural to draw the conclusion that the reduced hardiness of immature grain under storage conditions may possibly be due to its higher rate of respiration. It is well known that the respiration intensity of grain decreases according as its ripening is completed (1). But insufficiently ripe grain has a higher moisture content, too. What will be the increase in the respiration intensity of a less mature grain under moisture conditions equal to or approximating those of a ripe grain, has been but little studied as yet.

If treated in a due manner, the grain, before it is put loose for storage, should be reduced to a standard moisture content (14 per cent); it is therefore most important to find out what is the difference between the respiration intensities of grain of various ripeness, under identical or approximating moisture conditions, and especially in the condition of standard moisture.

We became able to answer this question after having studied active ventilation of newly harvested wheat grain of early and late hard dough (wax ripeness), a detailed report of which will be published elsewhere.

Grain of the wheat variety *Lutescens* 062 collected from the same plot and in the same stage of maturity was subjected to a comparative investigation, in order to determine its respiration intensity in consecutive stages of dehydration brought about by an air current of a definite temperature and humidity. The velocity of the air current was kept constant (12-fold exchange per hour) by means of calibrated rheometers (flowmeters). The respiratory gas exchange was measured by the manometric method, at a constant temperature (3) (see table below).

It may be seen from the table that: 1) under identical conditions of aeration grain of early hard dough attains a definite degree of humidity at a different date than grain of late hard dough; this is quite natural, since the initial humidity of the grain in these stages is different *. The second and decisive condition consists in that in the case of about the same humidity, the respiration proceeds much more intensely in a grain of early hard dough. And the difference is the more significant, the higher the moisture content in the grain serving for comparison. With a standard humidity and still lower moisture contents, the respiration intensity of a grain in the stage of early hard dough is twice as high as in a grain in the stage of late hard dough, or even more.

* In a grain of early hard dough is was equal to 28 per cent, in a grain of late hard dough to 18 per cent.

Moisture Content and Respiration Intensity of Wheat Grain of Early and Late Hard Dough

T° of air supplied to the grain by ventilation*	T° of respiration measurements	Early hard dough				Late hard dough			
		Length of ventilation, in days	Grain humidity, in %	Respiration intensity in mm³ per 10 g abs. dry matter		Length of ventilation, in days	Grain humidity, in %	Respiration intensity in mm³ per 10 g abs. dry matter	
				CO_2	O_2			CO_2	O_2
35	25	8	16,7	32	44	2	17,2	9	8
35	25	11	15,2	5	7	5	15,0	3	2
15	25	5	14,1	11	13	2	16,5	6	5
15	25	8	11,1	5	8	5	12,0	0,6	0
15	25	—	—	—	—	8	10,4	0,6	0

If the humidity of a grain in the stage of late hard dough is reduced to 10—12 per cent, its respiration becomes very weak, and can be detected by the evolution of CO_2 only. No oxygen respiration can be observed by the method of manometric measurements, even within a 24-hour exposure. Under the same moisture conditions, a grain at the stage of early hard dough displays a much higher intensity of respiration; both the evolution of CO_2 and the absorption of O_2 are clearly manifest thereby. The respiration coefficient of a grain of early hard dough is invariably below unity, whereas at the stage of late hard dough the grain has a respiration coefficient exceeding unity. This circumstance is very essential for the understanding of the varying hardiness of the grain to storage as dependent on its degree of ripeness, since the formation of water which accompanies respiration is due to the hydration of the oxygen absorbed from the air. The increased resistance to storage which characterizes a fully mature grain is due to the fact that the formation of water in the process of respiration proceeds at a lower rate because of a reduced oxygen absorption, especially under conditions of low humidity.

The respiration intensity of a grain in the stage of early hard dough and with a moisture content even below the conditional one (11 per cent) equalizes, if estimated by its oxygen absorption, the respiration intensity of a grain of late hard dough with a moisture content considerably surpassing (17 per cent) the one stipulated by the prescriptions for grain storage in granaries.

The spontaneous heating of dry grain, which has long since attracted attention both of farmers and students, and its resulting deterioration, may occasionally be due to the presence among the grain heaped for storage of not fully ripe grain, though dried in accordance with the corresponding regulations of the trade. Spontaneous heating of grain containing an admixture of immature grain is not excluded even if the conditions for a local overchilling of the bulk of grain are entirely eliminated. Overchilling is known ([4]) to call forth condensation of moisture on the grain, which accounts for all the harmful consequences of an increased humidity.

Inasmuch as for a variety of reasons harvesting is often done before due term, or else the crop to be stored contains much unripe grain from later stems (after-growth), such a grain, even if of standard moisture content, should be provided with optimal aeration, at least during the first months, until after-ripening is over. In the author's opinion, granaries supplied with an active ventilation system capable of eliminating moisture from the intergranular spaces in an immobile bulk of grain ([5]), are the best suited for securing a satisfactory preservation of insufficiently mature grain.

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Received
29. II. 1944.

REFERENCES

[1] Е. Леман и Ф. Айхиле, Физиология прорастания семян злаков, 1936. [2] J. Hoffmann, Das Getreidekorn, Berlin, 1912. [3] А. И. Смирнов и С. Д. Чигирев, Биохимия, 5, 358 (1940). [4] A. Kiesel, Vassilieva and Zygankova, C. R. Acad. Sci. URSS, XXIV, № 8, 786 (1939). [5] П. М. Дубинин, Активное вентилирование зерна, М., 1940; C. O. Swanson and Fenton, Agricult. Exp. St. Kansas, State Coll. of Agric. and App. Science, Techn. Bull. 33 (1932); C. F. Kelly, U. S. Dep. of Agric., Circular No. 544, Apr. 1940.

* Humidity of air supplied is 40 per cent in all cases.

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GEOLOGY

ON SOME PRINCIPLES OF CLASSIFICATION OF OIL AND GAS ACCUMULATIONS

By I. O. BROD

(Communicated by P. I. Stepanov, Member of the Academy, 7.II.1944)

The first seven decades of the existence of the world oil and gas industry are characterized by a development of the «structural theory» of the formation of oil and gas pools. The connexion between oil and gas accumulations and anticline folds was first pointed out in 1855—61 ([1], [2]). Within 1910 to 1930 a number of classification systems appeared, dealing with structural forms and oil and gas pools associated with them. The classification proposed by Clapp ([3],[4]) in 1910—11 and completed by him in 1917 and 1929—30 may be considered as summing up all the earlier work in this field. Clapp's classification, slightly modified, was also accepted by I. M. Gubkin ([5]).

Soon after the appearance of the «anticline» or, more correctly, «structural» theory of oil and gas fields, several papers were published, discarding or attaching much less importance to the rôle of the structural factor in the accumulation of oil and gas ([6-11]).

It was shown that the accumulation and distribution of oil and gas within a reservoir depends mainly on the character of the reservoir rock.

Wilson ([12]) was the first to propose to classify the types of natural oil and gas reservoirs instead of the form of pool structure. Wilson subdivided oil and gas reservoirs into closed and open ones. Closed reservoirs he subdivided into four groups.

1. Reservoirs closed by a local deformation of strata.
2. Reservoirs closed because of varying porosity of the rock.
3. Reservoirs closed by a combination of folding and of varying porosity.
4. Reservoirs closed by a combination of faulting and of varying porosity.

The period following the appearance of Wilson's classification was characterized in the USA by a detailed study of such natural reservoirs in which the accumulation of oil and gas is mainly associated with the lithological factor and stratigraphical relations between the reservoir and the overlying and underlying rocks. Reservoirs of this class are called by American geologists «stratigraphical traps».

Numerous papers by A. I. Levorsen ([13]) and some other writers have furnished abundant data on the mode of occurrence and accumulation of oil and gas in natural reservoirs which in this country became known as «stratigraphic accumulations» ([14]). These studies gave rise to classifications of «stratigraphic traps»([15]).

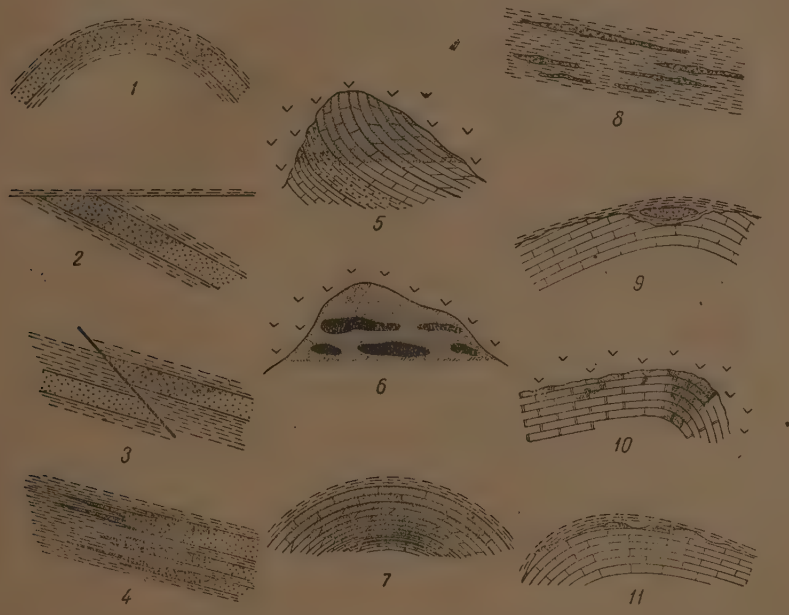
While «structural» classifications (Clapp and others) serve as a basis for discriminating between the different types of oil and gas pools, classifications of natural reservoirs (Wilson, Kornfeld, *etc.*) characterize the types of accumulations. In order to develop this statement, let us dwell upon the terms applied, namely: accumulation, natural reservoir, reservoir rock, and pool.

By the term «oil and gas accumulation» is meant the accumulation of oil and gas in a natural reservoir. In most cases the accumulation is limited by rocks of

low permeability, partly by water. Sometimes a reservoir may be bounded on all sides by impervious rocks.

A «natural reservoir» is a receptacle for oil, gas and water, wherein circulation of fluids may take place. Any natural reservoir consists of a reservoir rock, limited by rocks of low permeability.

A «reservoir rock» is a sufficiently permeable rock permitting the circulation of fluids. The form of a natural reservoir depends upon the relation between the reser-



1—4, bedded accumulations: *1*, arched; *2*, stratigraphically screened; *3*, tectonically screened; *4*, in wedging bed

5—7, massive accumulations: *5*, in an erosional projection; *6*, in a projection of a reef origin; *a*, zones of a high oil recovery: *7*, in a structural projection

8—11, accumulations of irregular shape: *8*, in lenses in a normal series of sedimentary rocks; *9*, in sand accumulations in depressions of the ancient relief; *10*, in zones of a high porosity and permeability; *11*, in projections of the ancient microrelief

voir rock and the impervious rocks containing it. Hence it follows that the form of an oil and gas accumulation corresponds to that of a reservoir only in case of its lithological limitation on all sides. Usually the reservoir contains mainly water, while the oil and gas accumulation occupies but a small part of it.

By «an oil and gas pool» are meant total accumulations of oil and gas within a certain area, controlled by a unit structural form.

When determining the types of oil and gas accumulations, three factors have to be taken into account:

1. The form of the fluid-containing reservoir.
2. The structure of the pool.
3. The relation and interaction of water, oil and gas within the reservoir.

The classification of oil and gas accumulations as suggested by the author in 1937 was based upon the type of reservoir which determines two principal groups of accumulations («bedded» and «irregular»), and then subgroups and types (species) of accumulations were distinguished in accordance with the deformation of the series, a part of which the reservoir forms ([16], [17]). Upon a revision of the data concerning the mode of oil and gas accumulation in various platform, foothill and

intramountain regions, the author arrived at the necessity of distinguishing not two but three principal groups of accumulations (see figure):

I. «Bedded accumulations», in which the reservoir rocks are of a bedded character, that is, are overlain and underlain by strata of a low permeability. The accumulation is restricted within the bed by edge waters.

II. «Massive accumulations», formed in thick beds covered by rocks of low permeability. Inside such a massif the distribution of the fluids takes place, in accordance with their specific gravity, irrespective of the mode of occurrence of the rocks, the oil and gas accumulation being limited by bottom water. The surfaces separating the fluids cut or intersect the body of the massif which represents a single reservoir from the standpoint of the formation of the accumulation.

III. «Irregular» or «lithological accumulations». The reservoirs of such accumulations are represented either by sand bodies of an irregular shape restricted by rocks of a low permeability, or by sufficiently porous and permeable zones within carbonate or crystalline rocks. In most cases such accumulations are lithologically limited on all sides. Sometimes within such a reservoir oil, gas and water occur together which in this case are distributed according to their specific gravity. The water underlying a lithological accumulation plays but a passive rôle. On the other hand, in bedded and massive accumulations water is the chief agent in the formation of an accumulation.

In his above cited work ([17]) the author distinguishes among the bedded accumulations, according to the type of dislocation, arched accumulations to a varying degree disturbed by faults, and accumulations tectonically or stratigraphically screened *. In accordance with the principles here exposed, accumulations in wedging beds should also be considered a subgroup of bedded accumulations.

Tectonically and stratigraphically screened accumulations, as well as those in wedging beds, may be formed only provided the gas and oil are propped up by water to the surface limiting the extension of the reservoir rock up its dip.

Massive accumulations should be subdivided according to the properties of the reservoir rock containing oil and gas. Usually a massif, representing unit reservoir as to the accumulation of oil and gas, during the process of exploitation disintegrates into several independent zones or horizons, differing in their recovery properties. This often occurs in the oil-bearing part of an accumulation. During the exploitation of gas caps or massive gas accumulations, due to the considerably greater mobility of gas, the whole massif will behave as a single accumulation. Massive accumulations may be associated with buried granite ridges, with limestone-dolomitic highs of erosive or reef origin, with limestone-dolomitic brachyanticlines and other tectonic uplifts. It is evidently the position of the zones of higher porosity within a projection that will account for the unequal oil recovery of a massive accumulation and depend upon the origin and formation conditions of the uplift.

Lithological oil and gas accumulations associated with reservoir rocks of irregular shape may be subdivided according to the character of the reservoir rock (say, a sandy and a carbonate one). Various types of accumulations are viewed in accordance with the relationship between the reservoir and the surrounding rocks; for example, clastic accumulations on an uneven surface of the ancient relief, coarse-grained lenses in fine sands or shales, zones of higher porosity in carbonate and crystalline rocks, weathering surfaces and saliences of the microrelief, *etc.*

All these principles of classification are accepted by the author for distinguishing all the known and possible types of oil and gas accumulations. The main factor accepted for this purpose is the type of reservoir (bedded, massive, and of irregular shape) containing oil and gas, and the relation of the fluids within the reservoir.

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* Screened accumulations are those occurring in beds brought up the dip in contact with rocks of low permeability either due to a fault (tectonically screened), or along the surface of unconformity (stratigraphically screened).

The structure produced by a local deformation of beds plays an important part only for bedded reservoirs.

The existing division of accumulations into structural and stratigraphic traps prevents from distinguishing clearly enough all the types of accumulations by considering the points of likeness and difference of groups, subgroups and types of accumulations. Such a division of accumulations has its historical grounds. At the beginning it was the structural factors of oil accumulation that attracted attention; later on, this gave place to stratigraphical factors.

The chief principles of classification of oil and gas accumulations, as suggested by the writer, are a further development of Wilson's classification. The present classification is based on the relative rôle of various geological factors explaining the relation between oil, gas and water within natural reservoirs.

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Received
7. II. 1944.

REFERENCES

[1] H. Oldham, Ceological report, London (1858). [2] T. Sterry Hunt, Canadian Naturalist, 6, 241 (1861). [3] F.G. Clapp, Econ. Ceol., V, 517 (1910); VI, 1 (1911); Bull. Geol. Soc. Amer., 28 (1917). [4] Idem, Am. Ass. Petrol. Geol., 2, 667 (1929); Pan. Am. Geol., 53, 29 (1930). [5] И. М. Губкин, Учение о нефти, 1937. [6] J. P. Lesley, Trans. Amer. Inst. Min. Eng., 14, 654 (1885—86). [7] M. J. Munn, Econom. Geol., 4, 509 (1909). [8] Chester Washburne, Trans. A. I. M. E., 50, 829 (1914). [9] A. W. Mc Coy, Journ. Geol., 27, 252 (1919); ibid., 24, 798 (1916). [10] К. П. Калицкий, Тр. Геол. комитета, нов. сер., в. 146 (1916). [11] Y. L. Rich, Bull. Am. Ass. Petrol. Geol., 8, 213 (1923). [12] W. B. Wilson, Problems of Petroleum Geol., p. 433, 1934; Pan. Am, Geol., 62, No. 5, 347 (1934); Bull. Am. Ass. Petrol. Geol., 26, No. 7, 1291 (1942). [13] A. I. Levorsen, ibid., No. 5 (1936); 27, 887 (1943). [14] М. Ф. Мирчинк, Стратиграфические залежи нефти, Баку, 1943. [15] Joseph A. Kornfeld, Oil Weekly, No. 6, 7 (1941). [16] И. О. Брод, Грозненский нефтяник, № 9 (1937). [17] И. О. Брод, Труды XVII. сессии Междунар. геол. конгресса, IV, 1941.

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PALAEONTOLOGY

A NEW FAMILY OF *COLEOPTERA* FROM THE PERMIAN OF THE URALS

By B. B. ROHDENDORF

(Communicated by A. A. Borissiak, Member of the Academy, 25.I.1944)

When identifying Permian insects collected in 1937 by E. V. Permiakova, near Chekarda on the Sylva river, I discovered remains of a peculiar beetle which I referred at first to the group of *Cupedoidea*. As we know, that superfamily of *Coleoptera* is represented at present but by some 20 species of a few genera of the relict family *Cupedidae*.

Fossil *Cupedoidea* are found from the Permian upwards, and had been doubtless much more numerous in the past than they are nowadays. However, a more detailed examination of the new fossil representative of *Cupedoidea* showed a great peculiarity of the latter; the structure of the wings sharply separates the new form not only from the known Permian *Permocupedidae* Mart., but from any other *Coleoptera* as well. The wings possess some characteristic, doubtless highly archaic features. All this points with certainty to an isolated position of this Permian beetle. I establish a new family for it

TSHEKARDOCOLEIDAE fam. nov.

Type of family: *Tshekardocoleus* gen. nov.

Venation of elytron more complete than in *Cupedidae* and *Permocupedidae*; main veins run more obliquely with regard to the long axis of elytron, not being parallel with fore or hind margin; anterior margin sharply divided off the rest, broad and flat, with a meshwork of veins; RS branching off at the junction of 1st and 2nd third of wing; apex of wing is reached but by SC, RS and sutural rib (An_3?); M with 2 long branches; these both, Cu and An_1 run parallel to each other and end at hind wing margin, often uniting with sutural rib (An_3?).

One genus: *Tshekardocoleus* gen. nov., from the Lower Permian, Middle Urals.

TSHEKARDOCOLEUS gen. nov.

Type of genus: *Tshekardocoleus magnus* sp. nov.

Anterior, costal margin with 2 rows of cells: SC situated close to R, being weaker than the latter; between these veins there is a row of cells in the vertical third of wing only; between RS_1 and RS_2, three rows of rather large irregular cells; both branches of M are parallel to each other from the 1st third of elytron: basal stem and branching of M approaching R; anterior branch of M not reaching hind margin, losing itself among small cells; posterior branch of M at its end runs parallel to the hind margin, uniting with the end of R; between hind margin (An_3?) and apical part of MP lies a row of cells: between the veins RS and MA, MA and MP, MP and Cu lie cells, 2 rows in each interspace. Legs short. Abdomen much shorter than elytra.

Single species: *Tshekardocoleus* sp. n.

Tshekardocoleus magnus sp. nova. Palaeontological Institute, No. 28/168, 29/168 (counter-part). Chekarda, left bank of Sylva. Middle Urals, 1937, E. V. Permiakova.

Length of elytron, 15.0 mm; width of same, 2.5 mm; total length of impression, 16.0 mm; probable total length of insect, 18.5—19.0 mm. Impression of 2 elytra, slightly diverging apically, part of pronotum, middle pair

of legs, and abdomen. Fossil coloured dark brown, abdomen almost black. Anterior margin of elytron and pronotum covered with numerous fine and very sharp pits. Remains of pronotum, legs and abdomen very imperfectly preserved.

The relations of the described Permian beetle with the rest of early *Coleoptera* are of great interest. The investigations of Tillyard (1924), Martynov (1932) and especially Richter (1935) threw light on this little known and difficult branch of palaeoentomology ([1-3]). The external resemblance of *Tshekardocoleus* to the true *Cupedoidea* seems to be only apparent; the described new form possesses highly archaic features, as *e. g.* the oblique course of the veins, the branching of M, the presence of a meshwork of veins on the costal area, the short abdomen. All this makes us consider *Tshekardocoleus* as a member of an early peculiar, probably extinct, group of *Coleoptera*. *Tshekardocoleidae* display some resemblance to *Cupedoidea*, on the one hand, and, in the presence of the 2nd branch of M, to *Sojanocoleidae*, on the other, this latter resemblance being, however, purely formal.

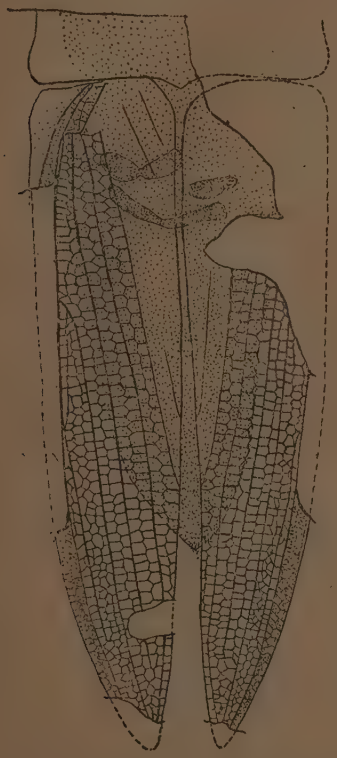

Fig. 1. *Tshekardocoleus magnus* gen. sp. n. Impression No. 28/168 of Palaeontological Institute

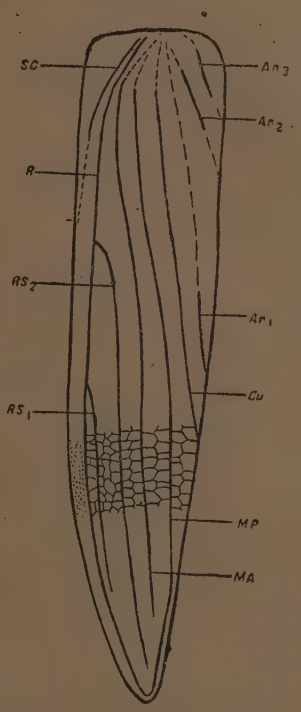

Fig. 2. *Tshekardocoleus magnus* gen. sp. n. Venation of elytron. Impression No. 28/168 of Palaeontological Institute. Denotations see in the text

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REFERENCES

[1] А. В. Мартынов, Труды Палеозоолог. ин-та, II (1932). [2] А. А. Рихтер, Энтомологич. обозрение, XXVI, № 1—4 (1935). [3] R. J. Tillyard, Proc. Linn. Soc., New South Wales, 49 (1924).

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GENETICS

PRODUCTION OF HIGHLY FERTILE TETRAPLOID BUCKWHEAT (*FAGOPYRUM ESCULENTUM*)

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It is known that low fertility is one of the peculiarities of experimental autopolyploids. This disappointing peculiarity invariably hindered the utilization of autopolyploid plants in agriculture, although otherwise they do favourably compare with diploids. This is especially evident for grain and, in particular, cereal crops where low fertility cannot be compensated for by any other properties. Being aware of all this, we decided nevertheless to make an attempt to obtain autotetraploids of typical cereal crops. *viz.* of buckwheat. We hoped to overcome the difficulty referred to above by making our work embrace a great number of hereditary variable forms. Therefore, we started our experimental work with a great number of buckwheat varieties at once (1). This paper contains the results of work with autotetraploid plants of the variety Bolshevik, bred by Pulman at the Bogorodizk Experiment Field (Kursk region).

Starting our work with the Bolshevik variety, our aim was to produce within it as large a number of polyploids differing as to their hereditary peculiarities. As a result, this variety was found to contain the largest number of initial autotetraploids obtained from 37 plants which under the action of cholchicine yielded autotetraploid seeds. In the next year, 1942, we planted the autotetraploid seeds from 25 of these plants, and ten most successful plants of the latter gave two generations within one year. All the material collected has been studied. By weighing unhusked and husked seeds and the husk and estimating the morphological characters of the seeds (shape and colour) we gained a clear idea of the seeds collected from each polyploid plant. These indices, compared with the data obtained on the percentage of abortive pollen and on the number of seeds collected from the same plants, made it possible to pick out the best specimens as regards large, plump grain and high yield. These best plants proved to be the progeny of eight initial tetraploid forms. The sowing material used in 1943 consisted only of seeds of the first and second generations of these initial forms.

Our judgment of the polyploid sowing of 1943 is based on a material embracing more than 1500 plants classed into four groups, according to the conditions of cultivation and to the sowing terms. The effect of these factors upon the development of the buckwheat is clearly visible in the table.

The plants of the first group were raised in a hothouse in hotbeds and in the garden of the Institute of Cytology in Moscow. Six families have produced a total of 350 tetraploids. In spite of their origin from five different initial forms and different sowing terms, all these tetraploids gave such similar results that we unite them all, as we unite also the data on the three diploid controls. In the first line, these six families showed an average weight unusually high even for polyploids (thousand seeds weighed on the average 40.9 g). It should be emphasized that the controls to these seeds also showed an unusually high mean weight of diploid seeds (29.4 g), surpassing considerably the weight of thousand seeds (23—25 g) adopted as the standard weight for the variety Bolshevik. We believe this increase in the average weight of the seeds to be due to regular watering, especially important for buckwheat, and to the richly dressed soil. The polyploids of the first group showed a rather low fertility, no more than $\frac{2}{3}$ of the fertility of the controls.

The inhibited development of the polyploid plants should be put down to the external conditions, since most of the material remained till the end of vegetation in pots where the vigorous polyploid plants stood much more closely than the diploid plants grown under similar conditions. Besides, the plants grown under Moscow conditions could not develop vigorously for lack of illumination they would enjoy in open fields.

An entirely different picture presents the material sown at three different dates on the territory of the Biological Station of the Institute of Cytology at Kropotovo near the town Kashira. Here all the three experimental groups and the two controls were sown close to one another in the open field, from 20 to 25 plants per 1 m². Under these conditions the buckwheat developed vigorously, showed profuse branching and attained the following heights: diploids, 110—120 cm; tetraploids, 150 cm and separate specimens even 170—180 cm.

The first Kropotovo group comprised 496 plants from 14 families which were the progeny of the first and second generations of a single initial form. This material was sown early (April 28) into boxes, kept in hotbeds, wherefrom a month later they were planted into the soil. The 35 diploid control plants of this group were inferior to this group of polyploids not only in seed weight, but also in fertility which were one and a half times higher in the tetraploids. As a result, the average yield of the new tetraploid forms was thrice that of the diploid plants (in grams per plant).

The second Kropotovo group, sown direct into ground on May 27th, gave still better results. It appeared that this term was the most favourable one of the three terms tried. The 408 test plants came from 10 families originating from plants of the 2nd generation, coming also from one initial form. The characteristics of the polyploids were excellent here. The average weight of the seeds was almost one and a half that of the controls. Moreover, the seeds collected from each tetraploid plant gave on the average a harvest almost four times exceeding that of the control. The average yield per tetraploid plant amounted to 21.85 g, whereas per diploid it was 3.85, *i. e.* the yield was increased by more than 5.5 times. It should be noted that separate record breaking plants gave up to 70 g of seed. The average minimum yield per plant was 331 seeds (family Б-25, 9 plants), and the maximum average (family Б-22, 44 plants), 779 seeds per plant *. In both cases the control and its average 161.5 remained far behind.

Less brilliant, but nevertheless satisfactory results were shown by the third Kropotovo group comprising 281 plants. Two families of this group originated

Place of sowing	Term of sowing	Tetraploids (4 n)					Diploids (2 n)					4 n in per cent to 2 n		
		Number of plants	Average number of seeds	Mean weight of seeds g/1000	Correlation coefficient	Mean weight of plant in g	Number of plants	Average number of seeds	Mean weight of seeds g/1000	Correlation coefficient	Mean weight of plant in g	Average number of seeds	Mean weight of seeds in g	Mean weight per plant
Moscow . . .	IV—V	350	96.0±3.3	40.9±0.3	+0.10±0.05	3.66	122	147.5±9.7	29.4±0.3		4.02	65.1	139.1	91.0
Kropotovo I	28.IV	496	180.4±6.8	37.6±0.2	−0.10±0.04	6.53	35	107.1±21.0	28.3±0.6	+0.31±0.15	2.18	168.4	161.4	299.1
» II	27.V	408	639.7±17.5	33.5±0.2	+0.25±0.05	21.80	35	164.4±29.9	22.5±0.5	+0.33±0.15	3.85	396.3	149.0	566.3
» III	12.IV	281	199.1±7.9	32.5±0.3	+0.29±0.05	6.73	—	—	—	—	—	—	—	—
		1535					192							

* The values of the coefficients of correlation between the average seed weight and the number of seeds per plant throughout the groups give no ground to speak of a negative coefficient, i. *e.* the weight of seeds does not increase at the expense of their number, and *vice versa*.

from the second generation of one initial form, while the third and fourth generations come from the first generation of the two other initial forms. We believe that the lower fertility of this group, as compared with the preceding one, should be accounted for by the late sowing term (June 12th), rather than by hereditary differences. There was no control to this last group *. Thus, a total of over 15 kg of tetraploid seeds from the Bolshevik sort were collected in 1943.

In order to give a full account of the new tetraploid form of buckwheat we should dwell here on the following features peculiar to it.

The polyploid stocks develop at a somewhat lower rate than the control, but they achieve their growing period by no more than 7 to 10 days later. This lateness is of no particular importance in buckwheat, and we duly collected the harvest from all the polyploid plants of the variety Bolshevik which, by the way, is a late ripening crop. A valuable economic character of our polyploids is that they do notlshed their seeds easily. The same has been observed by Arenkova for polyploid milet (2).

Overwintering tetraploid seeds are completely germinable. Tetraploid shoots, being more vigorous, germinate more readily and break the crust which is formed on thoe ground after rains more easily than do the shoots of the diploids.

N comparative observations on the drought resistance and winter hardiness of polyploid and normal plants of the Bolshevik variety are available. Several hundreds of Bolshevik tetraploids which remained on the field, easily withstood the three first morning frosts early in September, 1943.

Lastly, let us dwell upon the results of a small but very important experimene. Previous to breaking into blossom, one of the tetraploid plants was complettly isolated (family Б-17). In a pot the plant was taken away from the hothouse and placed under room conditions on the fourth story of a house in the centre of Moscow. Although the cross-pollination with other plants was utterly impossible, the open long-styled plant began setting fruit, and a total of 71 genuine seeds were collected. This shows that autopolyploids of buckwheat are capable of self-pollination. Self-pollination in usual diploid forms has been reported so rarely that there are writers (3) who consider self-pollination impossible for buckwheat.

Thus, we have succeeded to show on a sufficiently abundant material the possibility of experimentally producing autotetraploid forms surpassing in fertility the initial diploid forms. The cause of this success lies in the large amount of the initial material involved. The presence of initial tetraploids in 9 varieties of buckwheat made it possible to select the variety Bolshevik which gave the best response to polyploidy. It is very likely that the general background of the hereditary features of this variety was especially favourable for polyploidy. Within the variety Bolshevik, however, most of the initial forms proved also less fertile and displayed lower seed qualities. It was because of the great number of the initial autopolyploids within this variety that we were able to select individual plants endowed with the necessary qualities. Subsequent selection of the best plants among their progeny made it possible to obtain within a short period—two years and a half—new large-grained and, what is most important still, highly fertile forms of buckwheat. We are hopeful that these new forms will not only serve as a valuable material for subsequent breeding work, but that they will stand the test when sown to an industrial scale.

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REFERENCES

1 V. V. Saharov, S. L. Frolova, V. V. Mansurova, C. R. Acad. Sci. URSS, XLIII, No. 5 (1944). 2 D. N. Arenkova, ibid., XXIX, No. 4 (1940). 3 С. И. Коржинский и Н. А. Монтеверде, Труды СПБ об-ва естествоиспыт. 30, 1 (1900). 4 P. P. Richer, C. R., 5 (1904).

* The number of seeds collected from the test and control plants in Moscow and at the Kropotovo station is less than it might have been, for a great quantity of seeds were picked by small birds.

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